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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

L. H. DONNELL, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

March 1949 Vol. 2, No. 3

General Kinematics, Statics, Dynamics

(See also Revs. 298, 401)

284. **H. E. Merritt, "Gear trains,"** Pitman Publishing Co., New York, 1947. Cloth, 5.5 × 8.7 in., 178 pp., 48 figs., $6.

This book deals with two divisions of the subject of gearing, change-gear design and epicyclic or planetary gearing. The treatment of the design of change gears includes methods for simple and compound train design, with a table of primes and factors, and a Brocot table of decimal equivalents of ratios. The subject of epicyclic gearing is an extension of the material in the author's book *Gears*, and includes discussions of ratio, efficiency, and torque distribution, and concludes with analyses of tank and automotive differential gearing. H. C. Hesse, USA

285. **N. I. Levitski, "Synthesis of an articulated four-bar linkage with a given path of a point of the connecting rod"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, Oct. 1948, no. 10, pp. 1539–1542.

Given a curve, the author proposes approximately to determine a plane four-bar linkage $ABCD$ yielding this curve as the trajectory of a point M of the plane of the connecting rod BC. The problem is restated as follows: A is fixed, B moves on a circumference, M on the given curve, and the parameters are to be determined so that the sum of the squares of the deviations of C from a circle be a minimum.

Explicit formulas for the lengths AB, CD and the angle of AD with the x-axis are given in terms of the remaining dimensions, with the aid of summations of certain functions of these variables over the same positions of the mechanism as those used for the least square sums. A. W. Wundheiler, USA

286. G. J. **Talbourdet, "Motion analysis of several intermittent variable-speed mechanisms,"** *Trans. Amer. Soc. mech. Engrs.*, Jan. 1949, vol. 71, pp. 83–96, with discussion.

The following problem concerning the design of mechanical linkages is discussed in this paper: Given a shaft rotating at constant speed, to devise a linkage which will produce a regular intermittent rotation at a second shaft satisfying the conditions of negligible acceleration at the beginning and end of each cycle, and of smooth changes in angular acceleration. It is further assumed that the parts of the mechanism are rigid and fit accurately.

In general, the designs considered are based upon a planetary-gear system which adds together the input rotation of constant speed and an oscillating motion generated from the input motion. The designs differ in the methods employed for generating the required oscillatory motions. Detailed calculations are given for the following combinations: (*a*) internal planetary-gear system with rack and Scotch yoke; (*b*) fixed eccentric and sliding rack; (*c*) external planetary system and fixed eccentric; (*d*) internal planetary-gear system with rack or segment and cam; and (*e*) screw and gear with cam. In the case of a cam, a finite dwell time can be incorporated into the design.

In a published discussion of this article, J. A. Hrones points out the fundamental role played by planetary-gear systems in making such designs. A simple but adequate theory of this device is presented.

The material contained in this article should be of value to the designers of indexing devices. Horace M. Trent, USA

287. **John Aczél and Stephen Fenyö, "On fields of forces in which centers of gravity can be defined"** (in English), *Hung. Acta Math.*, 1948, vol. 1, no. 3, pp. 53–60.

The authors seek fields in which the field effect is rigorously equal to that on the total mass concentrated at the center of gravity, with an added condition that, for points on a straight line, the center is on the line. They find such fields necessarily parallel or central, with the magnitude of the forces in general proportional to the distance from a line in the plane, or from a plane in space. The special cases of constant parallel forces or central forces proportional to the distance appear.

Courtesy of Mathematical Reviews P. Franklin, USA

288. **P. Melchior, "On the dynamics of rigid bodies (Sur la dynamique des solides),"** *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 5, pp. 445–448.

The author finds the equations of motion of a rigid body in n-dimensional Riemannian space. In addition to the n co-ordinates of the centroid there are n^2 co-ordinates expressing the motion of the body relative to the centroid, and from the rigidity of the body there are $\frac{1}{2}n(n+1)$ invariants in the later co-ordinates. Expressing the kinetic and potential energies of the body in terms of the $n(n+1)$ co-ordinates and using Lagrangian multipliers of the invariants, the author applies Hamilton's principle and obtains, from a necessary reciprocal relation of the Lagrangian multipliers, the equations of motion relative to the centroid. In the particular case of a three-dimensional body it is shown that these are precisely the well-known Euler's equations.

P. C. Dunne, England

Gyroscopics, Governors, Servos

289. **John R. Ragazzini and Lotfi A. Zadeh, "Probability criterion for the design of servomechanisms,"** *J. appl. Phys.*, Feb. 1949, vol. 20, pp. 141–144.

In a servomechanism whose output is supposed to track the input, a measure of the performance is the error, or the difference between output and input for a specified test input. The design constants are usually so chosen as to minimize the error, one criterion being minimum rms error.

The authors examine another criterion in which there is maximized the probability that the error will not exceed a given maximum. By assuming the error to consist of systematic and noise components, design expressions are obtained by maximizing the probability function with respect to the design parameters. From these relations the parameters of the servo may be formally calculated. The authors derive similar expressions based on the

minimum rms error criterion; a comparison reveals that if either the noise or the systematic error is predominant, the two criteria are equivalent. Since minimizing the rms error leads to considerably less computation for the parameters, it should be used whenever applicable. Vincent Salmon, USA

Vibrations, Balancing

(See also Revs. 313, 359, 373, 401)

290. F. M. Dimentberg, "Method of 'dynamic stiffness' applied to the calculation of frequencies of systems with damping" (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, Oct. 1948, no. 10, pp. 1577–1597.

The author has generalized the dynamic stiffness, or dynamic modulus concept, to include viscously damped torsional oscillatory systems acted upon by a decremental moment, $M = M_0 e^{i(\omega+i\alpha)t}$. The complex dynamic stiffness vector, K, is obtained from that for an undamped system by substituting $-I(\omega + i\alpha)^2$ and $c + 2i(\omega + i\alpha)\xi$ for $-I\omega^2$ and c. Here c is shaft stiffness, 2ξ the viscous damping coefficient.

The systems analyzed include a built-in and a free system of one mass and one shaft, a free system of many flexibly connected masses, and a shaft with continuously distributed mass. The locus of the tip of the vector K for a constant α winds around the ω-axis in the space defined by the co-ordinate axes ω, $Re(K)$, $I(K)$, intersecting the $\omega - I(K)$ plane at points corresponding to the resonant and antiresonant frequencies.

The complex natural frequencies of a combination of two systems are obtained by graphical determination of the points at which $K_A + K_B = 0$, K_A and K_B being the complex dynamic stiffnesses of the systems referred to the junction point. Both the frequency and decrement of the free vibrations of the combined system are so obtained. Walter W. Soroka, USA

291. J. F. Shannon, "Vibration problems in gas turbines, centrifugal and axial-flow compressors," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2226, Mar. 1945 (issued in 1947), pp. 1–46.

The paper discusses the vibration problems involved in the operation of jet-propulsion engines, with emphasis placed upon turbine blades, the impeller vanes of centrifugal compressors and the blades of axial-flow compressors. The vibration of the combustion chambers and the disks is dealt with to a lesser extent. The numerous examples of turbine blade and compressor failures which are presented were studied under the author's direction by the Vibration Division of the Royal Aircraft Establishment.

High-frequency pressure fluctuations of the gas stream are cited as the chief source of excitation causing dangerously high vibration amplitudes when the frequency of the fluctuations coincides with a natural vibrational frequency of the component. The importance of the higher vibrational modes is emphasized. Stalling flutter of compressor blades is discussed and a rule presented for avoiding this condition; the rule is based on the relation of the thickness-chord ratio at the blade root to a quantity composed of the frequency parameter, fluid velocity and aspect ratio.

The distribution of damping for a specific light-alloy compressor blade and a specific turbine blade is presented. For the compressor blade the distribution was found to be: aerodynamic damping, 60 per cent; material damping, 17 per cent; and blade fastening damping, 23 per cent. For the turbine blade the distribution was found to be: aerodynamic, 16 per cent; material damping, 14 per cent; and blade fastening damping, 70 per cent. The total damping, δ, of the compressor blade was 0.042 and the total damping of the turbine blade was 0.043. Two methods of laboratory excitation of turbine and compressor blades are discussed. A moving-coil loudspeaker unit was used for the small excitation forces needed for compressor blades, and a pneumatic exciter for the larger forces required by turbine blades.

Richard H. Kemp, USA

292. R. C. Binder and G. G. Mize, "Strand vibrations in a roller-chain drive," *J. Franklin Inst.*, Jan. 1949, vol. 247, pp. 25–32.

An approximate analysis is made of the natural frequencies of vibration of roller-chain strand running between sprockets. The strand is assumed equivalent dynamically to a string in tension loaded with a mass at each roller; damping is neglected. Experimental results indicated that the fundamental mode occurred at a sprocket speed about 14 per cent in excess of calculated speed. Resonance occurred when the sprocket-tooth contact frequency coincided with a natural frequency of the chain span. For high-speed drives an added tension due to centrifugal force is suggested as a possible reason for lack of correlation in other experiments.

T. J. Dolan, USA

293. Fritz John, "On harmonic vibrations out of phase with the exciting force," *Commun. appl. Math.*, Dec. 1948, vol. 1, pp. 341–359.

The author shows that important general properties of a class of solutions of the differential equation $\ddot{x} = F(x, \cos \omega t)$, with $F(x, z)$ defined and of class C^2 in the strip $-\infty < x < +\infty$, $-1 \leq z \leq +1$, and with $F_x < 0$, $F_z > k > 0$ (k fixed), $\lim |F| = \infty$, $\lim F_z/F = 0$, for $|x| \to \infty$, can be obtained in a rigorous way, without any further assumption and hence also avoiding the usual assumption that the nonlinearity of this generalized basic equation of nonlinear vibration theory is only "slight."

The solutions considered are the "harmonics out of phase," which are solutions of frequency ω attaining maxima at the minima of $\cos \omega t$ and minima at the maxima of $\cos \omega t$ (such as occur in the linear case on the upper branch of the resonance curve). The stability of these harmonics is discussed (again in general terms only) with the help of the variational equation

$$\ddot{\zeta} = F_x(x, \cos \omega t)\zeta$$

where $x = x(t)$ is the harmonic under consideration. It is shown that certain properties of solutions of this equation are closely connected with the slope $d\omega/da$ (a = amplitude) of the response curve. J. H. Greidanus, Holland

294. B. Fraeys de Venbeke, "Characteristic phase shifts and forced vibrations of a damped system (Déphasages caractéristiques et vibrations forcées d'un système amorti)," *Bull. Acad. Belg. Cl. Sci.*, 1948, vol. 34, no. 7, pp. 626–641.

Expressions for the steady-state small oscillations of a damped system subjected to sinusoidal excitation are established in analogy to the well-known representation by normal components in the case of an undamped system. The equations of motion being given as

$$M[\ddot{q}] + R[\dot{q}] + C[q] = [p] \sin \omega t$$

where M, R, C are symmetric positive-definite square matrices, and [ ] denotes a column matrix, the steady-state solution has the form $[q] = [y] \sin(\omega t - \varphi)$ and $[y]$ is determined by

$$\{\tan \varphi (C - \omega^2 M) - \omega R\}[y] = 0.$$

The vanishing of the determinant gives the characteristic phase shifts φ_k in terms of ω; "characteristic modes of response" may be

defined by $[s_k] = R[y_k]\ (\omega/\sin\varphi_k)$, and it is shown that the $[s_k]$ are linearly independent among themselves and orthogonal to the $[y_j]$, if $j \neq k$; they can serve as a base for the given excitation $[p]$. The dependence of the φ_k on ω is discussed.

G. Kuerti, USA

295. Hans **Kleinwächter and Herbert Wojtech, "Graphical determination of transient phenomena of vibrating systems through complex representation (Graphische Ermittlung der Einschwingvorgänge von Schwingungssystemen mittels komplexer Darstellung),"** *Arch. elekt. Übertr.*, Feb.-Mar. 1948, vol. 2, pp. 69–75.

A graphical solution is given of damped-vibration problems which are governed by the equation $\ddot{y} + a\dot{y} + by = bf(t)$, in which $f(t)$ may represent a continuous or a discontinuous function of t, y and $\dot{y}$. If $f(t)$ is a continuous function it is approximated by a "staircase" curve. The method is essentially based on the well-known fact that, in the elementary case where $f(t)$ is sinusoidal, the resulting vibratory motion can be derived from a uniformly rotating vector the end point of which moves along a logarithmic spiral, by projecting this end point on the imaginary co-ordinate axis. An apparatus for drawing such a logarithmic spiral is described.

C. B. Biezeno, Holland

Wave Motion, Impact

296. Sokrates **Rombakis, "On the phase velocity of waves of finite amplitude (Über die Phasengeschwindigkeit von Wellen endlicher Amplitude),"** *Z. Met.*, July 1947, vol. 1, pp. 302–303.

In this note it is shown that the phase velocity of water waves of finite amplitude is given by the equation

$$c = [(g\lambda/2\pi)\ (1 + 4\pi^2 A^2/\lambda^2)]^{1/2}$$

where λ is the wave length and A is the amplitude of the wave. The analysis is based on the fundamental theory of G. G. Stokes.

Louis A. Pipes, USA

297. **W. Lenz, "Ship waves treated in elementary manner (Schiffswellen in elementarer Behandlungsweise),"** *Ann. Phys.*, 1947, vol. 1, nos. 1-3, pp. 75–82.

The ship waves are built up from plane wave systems and, after introducing the dispersion law, the special formulas of deep-water waves due to gravity and capillarity are worked out. The chief merit of the article seems to be a close conformity in the treatment of the gravity and the capillary waves, but the existing literature on the subject has not been fully considered, not even Lord Kelvin's early work. The phase difference at the boundary lines has not been mentioned or indicated in the figures, and by mistake an incorrect value of the angles between these lines and the axis of symmetry is given.

Einar Hogner, Sweden

298. **Wilbur L. Mayo, "Analysis and modification of theory for impact of seaplanes on water,"** *Nat. adv. Comm. Aero. Rep.*, no. 810, 1945 (issued in 1948), pp. 1–12.

An analysis of the available theory of seaplane impact, and a proposed modification thereto are presented. In previous methods the over-all momentum of the float and virtual mass was assumed to remain constant during the impact, but the present analysis shows that this assumption is correct only when the resultant velocity of the float is normal to the keel.

The proposed modification chiefly involves consideration of the fact that the forward velocity of the seaplane float causes momentum to be passed into the hydrodynamic down-wash (this is the only consideration in the case of the planing float), and consideration of the fact that, for an impact with trim, the rate of penetration is determined not only by the velocity component normal to the keel but also by the velocity component parallel to the keel, which tends to reduce the penetration.

Experimental data for planing, oblique impact, and vertical drop are used to show that the accuracy of the proposed theory is good.

Ernest G. Stout, USA

299. **Margaret F. Steiner, "Comparison of over-all impact loads obtained during seaplane landing tests with loads predicted by hydrodynamic theory,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1781, Jan. 1949, pp. 1–47.

Flight tests of an amphibian-type flying boat were made in order to determine the applicability of theory to the case of the conventional full-scale airplane. Landings were made in calm water for a range of angles of trim and flight path as wide as practicable. The loads and displacements obtained in smooth-water landings in which little rotation in pitch occurred and in which only the V-bottom portion of the forebody was involved, were in good agreement with computed values based on general hydrodynamic impact theory.

In the majority of the normal landings the chines were immersed in the free-water surface at the time of maximum load. Due to the lower dead rise on the curved chine region the local pressures on the region were considerably higher than in the region of the keel, which had an angle of dead rise of 20 deg. The resultant maximum loads for all cases of chine immersion were about 60 per cent greater than those computed for a V-bottom prismatic body having an angle of dead rise of 20 deg with no allowance made for transverse curvature at the chine.

In those impacts in which the aircraft rotated to a lower trim with a large angular velocity, the experimental loads in the early portion of the impact, in which only the V-shape portion of the forebody was involved, were considerably less than the computed loads, but there was no corresponding reduction in maximum load.

Ernest G. Stout, USA

Elasticity Theory

(See also Rev. 329)

300. **D. I. Sherman, "On the state of stress of some shrink-fit pieces"** (in Russian), *Bull. Acad. Sci. USSR Ser. tech. Sci. (Izv. Akad. Nauk SSSR Ser. tekh. Nauk)*, Sept. 1948, no. 9, pp. 1371–1388.

An analysis is given of the shrink-fit stresses of a hub of square cross section shrunk into a cylindrical shaft. The problem is approached as one of plane strain, and is treated by means of complex variables and conformal mapping. In order to map the square boundary into a unit circle, the sharp corners had to be replaced by radii such that the difference between the length of diagonals of the basic square and of the round-cornered figure is 11.6 per cent.

The mathematics is rather formidable, solutions being obtained in the form of series the individual terms of which, in turn, represent sums of subsidiary series. Numerical results are given for the contact stresses for various ratios of shaft diameter to side length of square hub. It appears from these results that for ratios smaller than 0.5 these stresses deviate but little from those of the ordinary cylindrical shrink-fit for the same ratio of shaft to hub diameter. At larger ratios, up to 0.95, very considerable stress concentrations result which attain their maximum values at the intersections of the diagonals with the shaft surface.

It is claimed that this method, with corresponding modification, can be applied to the case of a square shaft with a number of circular holes, being shrunk onto a corresponding number of shafts, a problem that is stated to have its application for arms of crankshafts. George Winter, USA

301. N. D. Tarabasov, "Determination of stresses in a thin plate with several pressed-in round disks" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Nov. 1, 1948, vol. 63, pp. 15–18.

By the use of the complex variable the author derives formulas for the stresses arising in a circular disk with a free outer boundary due to press-fitting into it several smaller disks of the same thickness, the centers of the smaller disks being evenly spaced on a circle which is concentric with the large disk.

The derivation of the formulas is based on work by N. I. Muskhelishvili which was published in his 1935 Russian book, *Some Problems of the Theory of Elasticity.*

M. Gololobov, Czechoslovakia

302. A. E. Green, "The elastic equilibrium of isotropic plates and cylinders," *Proc. roy. Soc. Lond. Ser. A*, Feb. 3, 1949, vol. 195, pp. 533–552.

This paper deals with a general method of solution of a notoriously difficult class of three-dimensional boundary-value problems in the classical theory of elasticity, i.e. the determination of the state of stress and deformation in a body bounded by two parallel planes and one or two cylindrical surfaces whose generators are perpendicular to the plane boundaries. Arbitrary normal and shearing tractions are applied to the lateral boundary, whereas the bounding planes are free from surface tractions. Two basic loading cases are distinguished according as the lateral loading is: (*a*) antisymmetric with respect to the median plane ("bending") or (*b*) symmetric ("stretching").

In each of the two loading cases, the author takes as a starting point the general solution of the governing equations in terms of three harmonic stress functions originally introduced in modified form by J. Boussinesq. For reasons of economy certain complex stress combinations are used. The author then constructs an infinite aggregate of solutions possessing the appropriate symmetry, each member of which leaves the bounding planes free from surface tractions. The three stress functions generating the nontrivial component solutions of this aggregate are of the form

$$\left.\begin{aligned} \phi_i(\alpha, \beta, z) &= \varphi_i(\alpha, \beta; \mu)\,[\sin \mu z/h \text{ or } \cos \mu z/h] \\ \nabla^2 \varphi_i - \frac{\mu^2}{h^2}\varphi_i &= 0 \quad (i = 1, 2, 3) \end{aligned}\right\} \quad \ldots\ldots (1)$$

where the eigenvalues μ are the roots of the characteristic equations

$$\left.\begin{aligned} &\cos \mu \text{ or } \sin \mu = 0 \text{ for } i = 1 \\ &\sin 2\mu \mp 2\mu = 0 \text{ for } i = 2, 3 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (2)$$

Here α, β are suitable curvilinear co-ordinates in the median plane $z = 0$ of the plate, and $2h$ is the plate thickness. The stress field obtained via the stress functions (1) is summed over the eigenvalues μ and then expanded alternatively in a Fourier series or a power series in z, the same type of expansion being applied to the lateral loading. The boundary conditions for the lateral boundary are subsequently met by an iterative process, and the final solution appears in double series form. The problems of convergence here involved are not considered in detail.

Using only a first approximation to the solution for case (*a*), the author discusses the bending of an infinite plate containing a circular hole, and obtains results which are in close agreement with those established by E. Reissner [*J. Math. Phys.*, 1944, vol. 23; *J. appl. Mech.*, 1945, vol. 12; *Quart. appl. Math.*, 1947, vol. 5] by a different method. A more refined solution of the same problem on the basis of the general method presented in the paper is contemplated, pending availability of numerical tables of Bessel functions of the second kind of complex arguments. In this connection it should be noted that the relevant roots of the second of Equations (2) are complex, giving rise to complex arguments of the characteristic functions $\varphi_i(\alpha,\beta;\mu)$ $(i = 2, 3)$ in Equations (1). This feature considerably aggravates the difficulties encountered in applications of the method to specific problems.

E. Sternberg, USA

303. Ken Ikeda, "On the plane stresses in orthogonal anisotropic plates having a circular or an elliptic hole" (in Japanese), *J. Soc. Sci. Culture Japan*, Aug. 1947, vol. 2, pp. 26–31.

The author applies the general expressions for plane stresses in orthogonal anisotropic plates, previously obtained by him [*J. Soc. aero. Sci. Nippon*, Oct. 1942, vol. 9, pp. 1209–1219; *Rep. aero. Res. Inst. Tokyo*, July 1943, vol. 19, no. 259, pp. 249–277], to plates having a circular or an elliptic hole under the action of uniform tension or uniform shear. In the case of a circular hole some numerical results are also given. S. Moriguti, Japan

304. Ken Ikeda, M. Uemura, and M. Muto, "On the plane stresses in anisotropic plates having a circular or an elliptic hole with uniform pressure or shearing force on its periphery" (in Japanese), *J. Soc. appl. Mech. Japan*, Jan. 1948, vol. 1, pp. 8–14.

Further applications are made of the same method as was used in previous papers (see the preceding review). It is shown that the so-called Wolf's assumption concerning the elastic constants of orthogonal anisotropic plates used by K. Wolf [*Z. angew. Math. Mech.*, Oct. 1935, vol. 15, pp. 249–254] and others, is adequate for solid wooden plates but not for plywoods.

S. Moriguti, Japan

305. N. Dewulf, "Mohr's circle and conics of elasticity (Cercle de Mohr et coniques de l'élasticité)," *Houille blanche*, Nov.-Dec. 1947, v 2, pp. 481–487.

The author of this article briefly explains the use of Mohr's circle, turns to the double conics of elasticity of Cauchy, and proceeds to a detailed discussion of the various cases.

Next, the author introduces another locus associated with the state of strain involving the ratio of direction strain to principal strain, and explains its possible uses. The article ends with a chart showing the various possible forms which this locus may take. M. Bricas, Greece

306. J. B. Diaz and H. J. Greenberg, "Upper and lower bounds for the solution of the first boundary value problem of elasticity," *Quart. appl. Math.*, Oct. 1948, vol. 6, pp. 326–331.

Knowing the displacements on the boundary of an elastic body, acted on by given external forces, upper and lower bounds are determined for the displacement of an internal point, with the help of two special displacement types one of which satisfies only the boundary conditions, the other only the system of Navier's equations.

Besides these displacements, a solution for a homogeneous system (with zero external forces) is introduced, as well as a tensor which coincides at the frontier with the regular part of Green's tensor. The bounds are obtained by means of some inequalities

equivalent to the principle of minimum potential energy or to Castigliano's principle, and by means of Schwarz's inequality applied to the quadratic part of the integral which defines the potential energy. Aurel A. Beles, Rumania

307. Teruyosi **Udoguti, "Some examples of solutions in two-dimensional problems of elasticity by bipolar co-ordinates. Part I. A semi-infinite plate containing a circular hole"** (in Japanese), *Trans. Soc. mech. Engrs. Japan*, May 1947, vol. 13, pp. 17–40.

Jeffery's method [*Phil. Trans. roy. Soc. Lond. Ser. A*, 1921, vol. 221, pp. 265–293] is applied to several problems. In the case of tension parallel to the straight boundary a mistake in Jeffery's calculation is corrected, and the result is thereby brought into good agreement with the results of photoelastic researches. Four other loadings are treated, including the cases of distributed load on the circular hole. Application to the design of riveted joints is suggested. S. Moriguti, Japan

308. **Teruyosi Udoguti, "Problems of plane stresses in a plate with two circular holes"** (in Japanese), *Appl. Math. Mech. (Ōyō Sūgaku Rikigaku)*, 1947, vol. 1: June, pp. 14–28; July, pp. 61–81.

Further applications of Jeffery's method (see the preceding review) are made by the author. Two circular holes having equal diameters are considered. Eight cases of loading are treated, some of which have already been treated by other authors. Elaborate calculations by means of infinite series are made somewhat easier by means of a technique for increasing the convergency. One of the cases treated coincides with the recent investigation of C. B. Ling [see APPLIED MECHANICS REVIEWS, March 1948, Rev. 411]. S. Moriguti, Japan.

Experimental Stress Analysis

309. **W. A. P. Fisher, "Basic physical properties relied upon in the frozen stress technique,"** *Proc. Instn. mech. Engrs.*, 1948, vol. 158, no. 2, pp. 230–235, 2 plates.

The author deals with the properties of Catalin 800 when used for three-dimensional photoelasticity. This material has a higher optical sensitivity than Bakelite BT 61-893 and is available in much larger sizes. Its modulus of elasticity, ultimate strength and softening temperature are lower, however. The author describes some applications in which the material was used satisfactorily, the necessary technique of curing having been followed. The "rind effect" produced during heating is still, however, a serious drawback in its use. A. J. Durelli, USA

Rods, Beams, Shafts, Springs, Cables, etc.

310. K. **Rühl, "Calculation of reinforcing rings (Berechnung von Versteifungsringen),"** *Bauplan. Bautech.*, Aug. 1948, vol. 2, pp. 239–243.

Formulas are developed for bending moments and circumferential forces in a thin circular ring loaded in its plane. Loads symmetric about a diameter are assumed. They are of the following four types: (1) concentrated force balanced by circumferential shear stresses distributed according to the Bernoulli-Euler theory of bending; (2) concentrated force balanced by two equal forces which are either parallel or directed radially; (3) concentrated force balanced by a linearly varying normal pressure; (4) self-equilibrating continuous load parallel to the axis of symmetry, of intensity directly proportional to distance from a diameter. These load distributions are those commonly assumed in design analyses of stresses in stiffening rings as used in fuselages and other cylindrical structures. The formulas developed should be useful to designers making the conventional assumptions.

P. S. Symonds, USA

Plates, Disks, Shells, Membranes

(See Revs. 302, 303, 304, 307, 308, 312)

Buckling Problems

311. **F. Müller-Magyari, "Critical stresses of thin-walled plate-structures under central load (Kritische Spannungen dünnwandiger Plattenwerke unter zentrischem Druck),"** *Öst. Ingen.-Arch.*, Sept. 1948, vol. 2, no. 5, pp. 331–346.

The author considers the buckling of thin-walled plate structures composed of narrow rectangles of equal lengths, joined rigidly along their parallel longitudinal edges. The load on each plate is assumed (*a*) to act in the median plane of the plate, (*b*) to be uniformly distributed over a segment of the plate width, being zero elsewhere, and (*c*) to remain in constant ratio throughout the loading process.

Two possibilities may occur: (*a*) Euler buckling of the structure as a whole and (*b*) local buckling of individual plates. This paper considers only the second type of deformation and at first a single plate with edges elastically built-in is discussed. The author establishes the critical values for different elastic-edge conditions by means of the differential equation for plates, and the results are given in the form of diagrams. In continuation the general case is considered, but certain simplifications are introduced for symmetrical and closed sections in order to apply the foregoing results. For open sections, the author shows that this method leads to very complicated solutions.

An approximate solution based on Trefftz's method for boundary problems closes this article. Aurel A. Beles, Rumania

312. **R. G. Chapman, "Compression tests on dural-balsa sandwich panels,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2153, June 1945 (issued in 1947), pp. 1–21.

Results of compression tests on 24 flat panels and 18 curved panels of dural-balsa sandwich construction are reported. It is found that the efficiencies of these panels are higher than sheet-stringer combinations of certain types. Due to poor bonding between the faces and the filling, the efficiencies of the curved panels are usually lower than those of the flat panels. Possible use of birch veneer to improve bonding qualities is suggested.

Conrad C. Wan, USA

Structures

(See also Revs. 310, 368, 369)

313. **C. Scruton, "An experimental investigation of the aerodynamic stability of suspension bridges"** (in English with summaries in French and German), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, Third Congress, prelim. publ., pp. 463–473.

The author studied oscillations of sectional models of suspended bridges in a wind tunnel (up to wind speeds corresponding to 100 mph; inclinations from -15 to $+15$).

Oscillations were either torsional, or vertical. Critical values of the velocity are those for which casual oscillations are maintained, notwithstanding damping. "Reduced velocities" have the same values for model and prototype, provided that the logarith-

mic decrements due to damping in still air of the two structures are in a certain ratio with inertia and dimensions.

A number of stabilizing features were determined with reference to pitching oscillations (no vertical oscillations could be excited): traffic lanes should be separated by open slots, truss-type deck stringers preferred to plate girder type, sidetracks mounted outboard. Different types of handrails were studied in order to break up the spanwise continuity of the airflow pattern.

D. Gentiloni Silverj, Italy

314. Sven Olof Asplund, "Influence functions for the angular deviation correction in suspension bridges" (in English with French and German summaries), *Publ. int. Ass. Bridge Struct. Engng.*, 1948, Third Congress, prelim. publ., pp. 415–422.

The author has investigated the effect of the angular deviation of the cable elements upon the truss moments in a single-span suspension bridge. This was accomplished by including an additional term, which takes account of the angular deviation, in the classical suspension-bridge formula of J. Melan. The modified formula has been solved by the use of iteration.

Several examples have been included in the paper. The angular deviation correction was found to be small. For example, the correction is about −6 per cent of the maximum moment at the fifth point of a single-span suspension bridge having a sag-length ratio of 0.12, if the flexibility $c = l\,[(H_w + H)/EI]^{1/2}$ is between 10 and 20.

Karl Arnstein, USA

Rheology (Plastic, Viscoplastic Flow)

(See also Rev. 411)

315. F. Levi, "On the states of elastic constraint of viscous origin (Sugli stati di coazione elastica di origine viscosa)," *R. C. Accad. Lincei*, 1948, ser. 8, vol. 4, sem. 1: Jan., pp. 65–70; Feb., pp. 197–201.

The author studies the problem of taking into consideration the time factor in the solution of problems of homogeneous elastic solids which exhibit a viscous behavior. In the first note he considers flow defined as the plastic deformation which the material undergoes under the action of stress maintained over long periods of time; in the second note he considers shrinkage and combined shrinkage and flow. He defines a "specific viscosity" and assumes it to decay exponentially with time, and to be proportional at each instant to the stress. The general equation of equilibrium, set up in terms of this quantity, is found from a minimum principle attributed to Colonnetti [*Scienza Costruzioni*, Einaudi, 1941]. This equation is discussed for simply and multiply connected regions.

In the case of flow the static effect of the viscous phenomenon is shown to be analogous to that of an elastic system with varying modulus of elasticity. In the case of combined flow and shrinkage the state of stress at any instant can be calculated by a formula derived in the second note. If it be assumed that for both flow and shrinkage the constants entering the exponential decay are the same, it is shown that the effect after infinite time is independent of this constant and thus independent of the form of the curves representing the two phenomena.

Edward Saibel, USA

316. J. H. Palm, "Stress-strain relations for uniaxial loading" (in English), *Appl. sci. Res. Sec. A*, 1948, vol. 1, no. 3, pp. 198–214.

The author discusses some of the mathematical formulas commonly used to express the relation between stress and strain as determined by a tensile test. He demonstrates that such relations cannot be fitted accurately by equations of the type $\sigma = K_1 + K_2\epsilon^n$ or $\sigma = K\epsilon^n$ where σ is the true stress, ϵ the natural strain, and K_1, K_2, K and n are constants dependent upon the stress-strain properties.

Data on cold-drawn wire indicating that the tensile strength approaches a limiting, or saturation, value as the degree of strain hardening is increased to large values are reviewed. These data, and the observation that the slope of the true-stress versus natural-strain diagram (the plastic modulus) decreases continuously up to maximum load, are used as a basis for assuming that the plastic modulus for uniaxial stress should continue to decrease beyond maximum load. A mechanism of strain hardening which satisfies this condition is assumed and the following equation is derived:

$$\sigma = \sigma_t - (\sigma_t - \sigma_e)\exp(-\epsilon/\epsilon_c)$$

where σ_t is a limiting value of stress, σ_e is a hypothetical yield stress and ϵ_c is a constant depending upon the stress-strain properties. As noted by the author, a similar equation is presented by E. Voce ["The relationship between stress and strain for homogeneous deformation," *J. Inst. Metals*, July 1948, no. 1135].

Methods for evaluating the above constants are also presented.

William Schroeder, USA

Failure, Mechanics of Solid State

(See Rev. 291)

Design Factors, Meaning of Material Tests

(See Rev. 318)

Material Test Techniques

(See also Rev. 291)

317. P. Bernard, "Piezoelectric recording of stress-strain diagrams in dynamic tests (Enregistrement piézoélectrique des diagrammes 'tensions-déformations' au cours d'essais dynamiques)," *Mémor. Artill. fr.*, 1948, vol. 22, no. 2, pp. 305–322.

The first part of this paper gives a qualitative summary of the theories advanced to explain brittle fracture. The second part presents the results of tests with steel specimens, using a Charpy pendulum, the stress-time and stress-strain curves being determined by means of a piezoelectric gage. Little experimental detail is given. Under the conditions of the tests (notched bar and dynamic loading), the elastic limit and the breaking load of the specimens exceed the static values by a factor of about 2.5.

R. M. Davies, Wales

318. Marcel Prot, "A new technique of material tests—Fatigue test under progressive load (Une nouvelle technique d'essai des matériaux—L'essai de fatigue sous charge progressive)," *Ann. Ponts Chauss.*, July-Aug. 1948, vol. 118, pp. 441–464.

In investigating the fatigue strength of a specimen, it is common practice to apply a periodically changing load with a constant amplitude and to determine the number of cycles after which breakage occurs. The results of a number of tests with the same material and various values of the amplitude of the load may be represented in the form of a curve, the horizontal axis carrying the number of cycles and the vertical axis the largest stress acting during one cycle. This curve resembles a hyperbola, its horizontal asymptote marking the fatigue strength of the material. A large number of tests, and thus a great length of time, is needed in determining this hyperbola.

The author suggests a fatigue test, during which the maximum value of the stress is gradually increased: $P_t = \alpha t$. The value of α is constant during each test, but different for each specimen. If a graph is made in which the vertical axis represents the maximum stress at the moment of failure and the horizontal axis the square root of the value of α, the results of all tests with a specified material are located on a nearly straight line. The intersection of this line with the vertical axis determines the fatigue strength.

After a theoretical explanation for this procedure the results of a number of tests are presented and show good agreement with the theory. It is stated that the accuracy of the method is very good and that the total time needed for the investigation of a specified material is reduced to one tenth. A discussion of the needed testing machines, the choice of the values of α and the way of testing are included. R. G. Boiten, Holland

Mechanical Properties of Specific Materials

(See Rev. 317)

Hydraulics; Cavitation; Transport

(See also Revs. 323, 330)

319. **Henri J. Putman, "Unsteady flow in open channels,"** *Trans. Amer. geophys. Un.*, Apr. 1948, vol. 29, pp. 227–232.

The author gives a summary of a little-known study of Massau ["Mémoire sur l'intégration graphique des équations aux dérivées partielles," *Ann. Ass. Ingén. Éc. Gand*, 1900, vol. 23, p. 95]. He treats the characteristics of the differential equations of unsteady flow in channels, the initial and boundary conditions, and the formation and propagation of shock waves.

A. Craya, France

320. **Gerald P. Marcy, "Pressure drop with change of phase in a capillary tube,"** *Refrig. Engng.*, Jan. 1949, vol. 57, pp. 53–57.

This paper presents a new method for computing the length of a capillary tube to be used for the adiabatic expansion of a refrigerant. The rate of flow for a given tube may be found also. The solution is obtained by a graphical integration of the Fanning equation and does not depend upon empirical factors. Good correlation with test data has been found within the limited range of conditions investigated. R. C. Binder, USA

321. **J. S. Meyers and E. A. Schultz, "Tidal currents,"** *Proc. Amer. Soc. civ. Engrs.*, Apr. 1948, vol. 74, pp. 501–520.

As one of the papers in the symposium "Panama Canal—The Sea-Level Project," this paper presents the results of an analysis and a model study of the tidal currents in the open sea-level canal. The study was based on a 60 × 600-ft canal section some 35 miles long including the Atlantic and Pacific entrances. A 20-ft maximum tide at the Pacific end and a 2-ft Atlantic tide preceding the Pacific tide by three hours were used for the velocity studies. A simplified analysis based on steady flow between the maximum differences in water level yielded a maximum velocity of 4.6 knots.

A detailed step analysis resulted in a picture of the tidal velocities in the canal as a function of time and location, with a maximum velocity of 4.5 knots. A 1:100 undistorted scale-model study of the currents yielded a maximum velocity of 4.4 knots, together with time-position-velocity information in excellent agreement with the analysis. The possibility of using tide regulating structures to reduce the currents is discussed and brief results of model studies on the use of such devices are presented.

In a discussion of this paper General Pillsbury [*Proc. Amer. Soc. civ. Engrs.*, Sept. 1948, pp. 1228-31] presents the basis of the detailed analytical method used to determine the tidal currents. He notes that the currents in an uncontrolled canal would vary widely each week and from week to week and it is not feasible to time the transits of large traffic to avoid the maximum currents. Therefore, the inclusion of tidal locks is advisable. Wm. Allen and B. A. Bakhmeteff [*Proc. Amer. Soc. civ. Engrs.*, Dec. 1948, pp. 1511–17] also discuss the approximate and more exact methods of determining the tidal currents. K. C. Reynolds [*Proc. Amer. Soc. civ. Engrs.*, Sept. 1948, pp. 1248–1251] critically discusses the methods used to obtain the probable roughness coefficient of the channel as well as the details of model test procedure.

J. M. Robertson, USA

322. **P. Alméras, "Some problems concerning the filling of locks for navigation (Quelques problèmes posés par le remplissage des écluses de navigation),"** *Houille blanche*, Sept.-Oct. 1948, vol. 3, pp. 408–415.

An increase in the speed with which traffic may be passed through a lock requires an increase in the speed of moving the vessel into the lock, an increase in the rate of filling and emptying the lock or an increase in the rate at which the ports may be opened and closed. In considering the means by which the lock may be filled more rapidly the author discusses arrangement of conduits and ports through which the water flows. Equations are set up for the discharge through a conduit terminating in a single port and conduits containing a series of ports along their lengths. Increased spacing between ports at the lower end of the conduits is recognized as one means of equalizing discharge through the ports. Curves are presented to show the qualitative effect of multiple ports on discharge. Glenn Murphy, USA

Incompressible Flow: Laminar; Viscous

(See also Revs. 297, 319, 322, 336, 352, 354, 355, 356, 363, 367, 376, 414)

323. Victor L. Streeter, "Fluid dynamics," McGraw-Hill Book Co., New York, 1948. Cloth, 9.2 × 5.6 in., 263 pp., 139 figs., $5.

Fluid Dynamics is a textbook of the mathematical theory of fluid motion for the advanced student of engineering or science. However, the many theorems, applications and examples provide useful reference material as well. The author has taken particular care to present both the background and the details of derivations in such a manner that a reader without training beyond the calculus can follow the developments. Consequently, introductions to several phases of mathematics are offered in addition to the theory of fluid flow. Examples which are illustrative of practical applications assist the reader in bridging the gap between theory and practice.

A major part of the book is devoted to the treatment of irrotational flow, but the elements of viscous flow and boundary-layer development are also presented, as well as a discussion of fluid shear, non-Newtonian fluids, and plastics. The contents are indicated by the following chapter titles and lengths: (1) Fluid Flow Concepts, 6 pp; (2) Fundamentals of Frictionless Fluid Flow, 24 pp; (3) Theorems and Basic Flow Definitions, 22 pp; (4) Three-Dimensional Flow Examples, 32 pp; (5) Applications of Complex Variables to Two-Dimensional Fluid Flow, 16 pp; (6) Two-Dimensional Flow Examples, 36 pp; (7) Blasius Theorem—Flow Around Cylinders and Airfoils, 19 pp; (8) Schwarz-Christoffel Theorem—Free Streamlines, 26 pp; (9) Vortex Motion, 26 pp;

(10) Equations for Viscous Flow, 16 pp; (11) Examples of Viscous Flow, 17 pp; (12) The Boundary Layer, 18 pp.

Fluid Dynamics is one of the McGraw-Hill Publications in Aeronautical Science edited by J. C. Hunsaker.

John S. McNown, USA

324. J. B. Bratt, K. C. Wight, and A. Chinneck, "Free oscillations of an aerofoil about the half-chord axis at high incidences, and pitching moment derivatives for decaying oscillations," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2214, Sept. 1940 (issued in 1948), pp. 1–14.

All tests refer to a wing of aspect ratio 4.44, with 9-in. chord and a symmetric Joukowski profile of 0.15 thickness-chord ratio. Values of the frequency parameter ω vary from 0.48 to 1.05; those of Reynolds number are 1.42×10^5 and 2.83×10^5. Stability appears to depend markedly on mean incidence α and on initial angle of deflection θ_0, the region of instability in the α, θ_0 plane being greatest for small values of ω. For $\omega = 0.48$ instability occurs independent of θ_0 for α between 13 deg and 19 deg ("critical" incidences), beyond this range some minimum deflection is necessary. The quoted lower critical incidence is less than the stalling angle.

Records are analyzed on the usual assumption of linear relationship between aerodynamic moments and the deflection angle. Results agree satisfactorily with direct measurements of moments, carried out earlier with a magnetostriction stress indicator in cases of forced oscillation. J. H. Greidanus, Holland

325. Jean Luneau, "On the effect of the inertia of the wake for a body moving in a fluid with a uniformly accelerated motion (Sur l'effet d'inertie des sillages de corps se déplaçant dans un fluide d'un mouvement uniformément accéléré)," *C. R. Acad. Sci. Paris*, Oct. 27, 1948, vol. 227, pp. 823–825.

The author has measured the steady and nonsteady drag of a circular disk moving along its axis. He finds that for a given acceleration the increment of acceleration drag over steady drag increases with increasing speed. He attributes this increment to the "inertia of the wake" and, in effect, computes the apparent mass variation. Stanley Corrsin, USA

326. A. R. Manwell, "Aerofoils of maximum thickness ratio for a given maximum pressure coefficient," *Quart. J. Mech. appl. Math.*, Dec. 1948, vol. 1, pp. 365–375.

The symmetric airfoil profile which in incompressible or subsonic compressible flow has the largest thickness/chord ratio for a given maximum pressure coefficient is determined. It is shown that in two-dimensional incompressible flow the solution is that previously obtained by Riabouchinsky. For small thickness/chord ratios this theoretically optimum body is closely approximated by an ellipse. The methods advanced are employed to obtain the von Kármán-Tsien solution, and some notes are included on the possible application of an electrolytic tank to obtain solutions for any pressure-density law. It is shown that the methods of the report can be extended to other similar problems; in particular, the axially symmetric flow. H. Julian Allen, USA

327. Tomijiro Moriya, "The theory of a lattice composed of airfoils of arbitrary profile form" (in Japanese), *J. Soc. appl. Mech. Japan*, Mar. 1948, vol. 1, pp. 41–47.

The conformal transformation by means of which the flow past a lattice of straight-line profile airfoils is transformed into the flow around a circle [W. F. Durand, *Aerodynamic Theory*, vol. 2, pp. 91–96] is already known. The author extends the solutions to the case of arbitrary airfoil sections, by adding an inverse-power series to the transformation function, and determining by successive approximations the coefficients such that the contour of the section is transformed into the circle. The pressure distribution over the section is expressed in terms of the coefficients of the series. No numerical example is worked out.

Itiro Tani, Japan

328. Mitsutoshi Kawaguchi, "An approximate solution for viscous flow at low speeds. I—Flow past a circular cylinder; II—Flow past a sphere" (in Japanese), *Rep. Inst. Sci. Technol. Tokyo*, 1948, vol. 2: Mar.-Apr., pp. 33–37; May-June, pp. 66–71.

An approximate method of solution is proposed for the viscous flow past a body of Reynolds numbers somewhat greater than 10, to which the approximations of Stokes or Oseen can no longer be applied. The method consists in (1) expressing the stream function by an appropriate series with eight constants; (2) determining the constants by making the velocity components vanish and the equation of motion hold at the body surface; and (3) satisfying the integrals of the equation of motion throughout the field of flow.

Drag experienced by the body, and streamlines and pressure distribution around the body are calculated and compared with the existing experimental data. Fairly satisfactory agreement is found at Reynolds number $R = 10$ to 20 for the circular cylinder, and at $R = 10$ to 70 for the sphere, the diameter being used in defining R. Itiro Tani, Japan

329. Bernard Epstein, "A method for the solution of the Dirichlet problem for certain types of domains," *Quart. appl. Math.*, Oct. 1948, vol. 6, pp. 301–317.

The purpose of this paper is to present a method for solving the Dirichlet problem for a class of domains which may be regarded as limiting cases of those domains for which the so-called alternating procedure of H. A. Schwarz is applicable. The domain S studied in this paper is the sum of two domains which are adjacent (rather than overlapping as in the Schwarz method) and for each of which the solution of the Dirichlet problem is known. The problem is to find the values of the desired function at the points common to the boundaries of the two component domains, for the complete solution could then be found by solving Dirichlet problems for each of the latter.

The plane domain S is of finite connectivity and its boundary is composed of a finite number of smooth arcs. It is supposed that by addition of a finite number of smooth arcs to the boundary the domain S is divided into two adjacent domains whose Green's functions are known. The author shows that the function which gives the values of the solution on the added arcs satisfies an integral equation of the second kind. In the case in which the kernel of this integral equation satisfies a certain inequality which we shall call A, the author proves that the integral equation has an unique bounded solution, and he gives a method of successive approximations in order to obtain this solution: the first approximation, the starting point of the procedure, is an arbitrary, continuous and bounded function. The author establishes inequalities which permit to evaluate the upper limit of error made during each stage of approximation. Finally it can be shown that inequality A, which the kernel must satisfy, holds if certain restrictions are imposed upon the given domain and on the added arcs.

The following particular case is then considered: the domain S consists of the entire plane except for a finite number of collinear slits, which constitute the boundary of S. In this particular case the inequality A holds. The author applies his method to two

simple examples which belong to the above particular case and the exact solutions of which are known. He takes several initial approximations and the subsequent approximation is determined in each case. The comparison with the exact solution shows that the results are very satisfactory. Ratip Berker, Turkey

330. **R. G. Folsom and C. K. Ferguson, "Jet mixing of two liquids,"** *Trans. Amer. Soc. mech. Engrs.*, Jan. 1949, vol. 71, pp. 73–77.

The problem of mixing or blending in a tank of two or more liquids of low viscosity and the same density has been investigated in terms of available information on turbulent fluid jets discharging into a very large volume. Under the assumption that all fluid inducted into the jet is mixed completely, the effectiveness of any arrangement has been expressed in terms of a performance parameter which is defined as the discharge in gallons per minute inducted per horsepower supplied. The calculated results for a large gasoline-blending tank demonstrate the superiority of performance of the propeller-type mixer over typical jets produced with transfer-pump and pipe-line equipment. The economics of the installation and operation of equipment were not included.
R. C. Binder, USA

Compressible Flow, Gas Dynamics

(See also **Revs. 319, 326, 367, 372, 375, 377, 379, 397**)

331. **Harry E. Murray, "Comparison with experiment of several methods of predicting the lift of wings in subsonic compressible flow,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1739, Oct. 1948, pp. 1–33.

Semiempirical formulas from several sources are given for predicting variation of finite wing lift-curve slope $dC_L/d\alpha$ with Mach number M, sweepback angle Λ (of quarter-chord line), and section thickness.

The formulas developed are compared with each other and with experiment; that including the thickness effect seems to give the best results, though it appears to be restricted to constant section-thickness parameter. Moreover, its use requires computation of the Kaplan factor. It is hardly necessary to add that results from all of those methods begin to diverge from experiment rapidly as the transonic region is approached. William Pell, USA

332. **Kazuo Kondo, "Lorentz transformation in aerodynamics"** (in Japanese), *J. Soc. appl. Mech. Japan*, Mar. 1948, vol. 1, pp. 57–63.

When the disturbances caused by the lifting system are assumed to be small, both the velocity potential and acceleration potential are determined by an equation of the same form as the classical wave equation, the velocity of propagation being the (constant) velocity of sound. The object of the paper is to translate into the language of aerodynamics the well-known properties of the wave equation. It is pointed out, for example, that the Lorentz contraction corresponds to the Prandtl-Glauert relation between compressible and incompressible flows. The problem of time and causality has also an analog there. Itiro Tani, Japan

333. **W. J. Strang, "A physical theory of supersonic aerofoils in unsteady flow,"** *Proc. roy. Soc. Lond. Ser. A*, Dec. 7, 1948, vol. 195, pp. 245–264.

A linearized theory for the effects of transient disturbances on an airfoil in two-dimensional supersonic flow is presented. An infinite line source of disturbances is integrated to give the velocity potential, φ, which is required to satisfy the fundamental wave equation for the propagation of sound waves. The emission from the sources is treated as a step function. Solutions of the integral equations for φ are obtained for both stationary and moving-line sources for the supersonic case. When the method is applied to compute the pressure distribution on an airfoil entering a sharp-edged gust or on an airfoil after a sudden change of incidence, two or more fields of flow are superimposed.

Pressure-growth functions and lift-growth functions are calculated and are shown graphically for the idealized case of an infinitesimally thin airfoil penetrating a sharp-edged gust at Mach numbers from 1.25 to 2. The values are divided by the Ackeret value for the steady-state conditions, thus giving a useful comparison. The pressure and lift-growth functions for the case where the airfoil undergoes a sudden change of incidence are also derived. Application of the method to the problem of flutter is made and the flexural damping derivative is obtained. Finally the method is extended to compute the lift of wings entering gusts for different values of wing loading and for several assumed gust profiles. F. K. Hill, USA

334. **Lester L. Cronvich, "A numerical-graphical method of characteristics for axially symmetric isentropic flow,"** *J. aero. Sci.*, Mar. 1948, vol. 15, pp. 155–162.

Detailed rules of procedure are given for applying the method of characteristics, using a combination of graphical and numerical methods to problems of axially symmetric isentropic flow. The physical characteristics are constructed through solution of approximate finite difference equations relating the fluid properties at the lattice points of the characteristics net.

As an illustration, the flow around a conical-nosed body is calculated. For this problem there is worked out a procedure for determining the form of a curved shock wave when the variations in entropy downstream of the wave are negligible.
Ascher H. Shapiro, USA

335. **R. M. Conard, "Effect of sweepback applied to airfoil surfaces: Part III, IV (L'effet de flèche appliqué aux aérodynes: Troisième Partie—Aéroélasticité et structure de l'aile en flèche; Quatrième Partie—Extension des propriétés de la flèche à d'autres éléments),"** *Rech. aéro. Paris*, Sept.-Oct. 1948, no. 5, pp. 3–14.

Part III of this series contains Chapters 7 to 9. In Chapter 7 various methods for the calculation of the spanwise lift distribution on a sweptback wing are outlined; experimental determinations are also considered. The qualitative effects of the amount of taper, of warping, and of sweepback on the aerodynamic properties of a wing are treated singly.

In Chapter 8 the distribution of forces on a sweptback wing is compared with that for the straight wing, to point up new factors which must be considered in structural design. The problems of aeroelasticity and of flutter for the sweptback wing are considered. These structural, divergence, and vibration problems are summarized in tabular form, and suggestions for their solution are given.

Chapter 9 is an extensive bibliography for Chapters 7 and 8.

Part IV is concerned with an extension of the properties of sweepback to the tail surfaces and to propellers. In conclusion, the over-all advantages and disadvantages of sweepback are assessed. J. S. Isenberg, USA

336. **R. M. Conard, "Effects of sweepback applied to airfoil surfaces: Part II (L'effet de flèche appliqué aux aérodynes: Deuxième partie—l'aérodynamique de l'aile en flèche aux grandes vitesses),"** *Rech. aéro. Paris*, July 1948, no. 4, pp. 51–66.

The stated purpose of this engineering study of the effect of sweepback is to analyze the present knowledge of the subject, with the aim of selecting the principal problems which have not yet received a definite solution, and to establish a research program for these problems. The work is in six parts, Part II containing Chapters 4 to 6. [Part I, *Rech. aéro. Paris*, May 1948; see APPLIED MECHANICS REVIEWS, January 1949, Rev. 105].

Chapter 4 deals with the aerodynamics of the sweptback wing at high speeds, both theoretical and experimental. The sweptback wing is analyzed alone, and then the stability and maneuverability of conventional aircraft equipped with sweptback wings are discussed. Also considered are tailless aircraft and their limitations, and delta wings.

In Chapter 5, the gaps in our aerodynamical knowledge of sweptback wings in each of the regions of high subsonic, transonic and supersonic flow are pointed out. The principal problems are presented in tabular form, and suggestions for their solution are given.

Chapter 6 is an extensive bibliography of the subject matter of this part. J. S. Isenberg, USA

337. Isao Imai, "Hodograph method for compressible flow" (in Japanese), *Appl. Math. Mech. (Oyō Sugaku Rikigaku)*, Sept. 1947, vol. 1, pp. 147–161.

A general method for solving the equations of two-dimensional potential flow of nonviscous gas, whose density ρ varies with pressure p following the relation $p = f(\rho)$, where f is an arbitrary function of ρ, is developed by applying the hodograph method. In particular, the exact solution for the adiabatic flow is obtained by using hypergeometric functions.

As a numerical example, the author treats Kirchhoff's flow, when a flat plate is placed normal to the flow and a dead-air region exists behind it. The general formula for the resistance coefficient C_D is derived for the gas obeying the law $p = f(\rho)$. For adiabatic flow, the author gives the formula,

$$2\pi/C_D = \pi + 4 - 2M^2 - a_2M^4 - a_3M^6 - a_4M^8 - \ldots,$$

which is exact to the term including M^8. Here M is the Mach number and $a_2 = {}^1/_2 + (\gamma + 1)B$, $a_3 = {}^1/_4 - {}^1/_2(\gamma + 1)(2\gamma + 1)B + {}^1/_2(\gamma + 1)(2\gamma + 3)C$, $a_4 = 5/32 + {}^1/_4(\gamma + 1)(3\gamma^2 + \gamma - 1)B - {}^1/_4(\gamma + 1)(2\gamma + 3)(3\gamma + 1)C + {}^1/_{16}(\gamma + 1)^2D + {}^1/_4(\gamma + 1)(\gamma + 2)(3\gamma + 4)E$, where γ is the specific heat ratio and $B = {}^1/_3 + \pi/_{12} - ({}^1/_3)\log 2$, $C = -3 + ({}^7/_8)\pi$, $D = -({}^1/_9)(\pi - 1)^2 + {}^4/_9 \log 2$, $E = -{}^{26}/_{15} + \pi/_{60} + ({}^{32}/_{15})\log 2$. For air ($\gamma = 1.4$) $2\pi/C_D = \pi + 4 - 2M^2 + 0.2898\,M^4 - 0.0028\,M^6 - 0.0044\,M^8$.

Humio Tamaki, Japan

338. A. D. Young, "Skin friction in the laminar boundary layer in compressible flow," *Coll. Aero. Cranfield Rep.*, no. 20, July 1948, pp. 1–31.

A general semiempirical formula is developed for the skin friction on a flat plate at zero incidence, both with and without heat transfer, in a uniform flow of a perfect gas with constant specific heats. This formula is derived from an approximate solution of Crocco's integral equation for the viscous stress. Tables and curves are included comparing the results obtained from this equation with those obtained by Crocco and other investigators. The problems of heat-transfer dissipation and the effect of radiation on the temperature measured by a thermometer are briefly discussed in the light of this formula.

The formula is then applied to the development of an approximate method for solving the momentum equation of the boundary layer of a cylinder without heat transfer.

The method also indicates a rapid forward movement of separation of the boundary layer with increase of Mach number at the leading edge, for constant ratio of velocity fall per unit length of plate to initial velocity. Nicholas Di Pinto, USA

339. E. D. Kane and R. G. Folsom, "Problems and progress in low-pressures research," *J. aero. Sci.*, Jan. 1949, vol. 16, pp. 46–54.

It is essentially at low densities that gas flow shows effects due to the fact that the medium can no longer be regarded as continuous. The important factor is the ratio of the length of the mean free path L to a characteristic linear dimension D. When L/D exceeds 0.001, "slip" at solid boundaries ensues; when L/D greatly exceeds unity, collisions of molecules with the object are much more frequent than with one another. At pressures (densities) higher than those where slip occurs, increased boundary-layer thickness occurs, while at lower pressures a shock can no longer be regarded as practically discontinuous.

Experimental techniques include telemetering high-altitude rockets, models moving in low-pressure vessels, low-pressure wind tunnels, molecular beams, and high-speed rotating cylinders. Instrumentation is a major difficulty. Impact pressures can still be measured at low densities, but some additional quantity is necessary. Temperature measurement is difficult, and direct measurement of static pressure may involve prohibitive time lag. Flow visualization is being attempted by the use of the monochromatic absorption of oxygen and other gases, and the afterglow in excited nitrogen may prove utilizable.

Two small low-pressure wind tunnels at Berkeley, Calif., have given results which are being used in the design of a much larger one, with a working section of about 150 sq in., in which models with a characteristic length of about one inch can be tested. Diffusers have been found to be very inefficient, apparently because of boundary-layer thickness, and this must also be taken into account in nozzle design aimed at producing a larger area of uniform speed over the section.

Molecular beams from a furnace, collimated by slits, are being studied, and theoretical examination indicates that results may be expected from high-speed rotating cylinders.

W. G. Bickley, England

340. K. O. Friedrichs, "Formation and decay of shock waves," *Commun. appl. Math.*, Sept. 1948, vol. 1, pp. 211–245.

Consider a tube which contains a gas initially at rest. At one end of the tube is a piston. The present paper is concerned with the formation and propagation of shocks which are formed when the piston is moved along the tube. Of particular interest is the case where a rarefaction wave formed by stopping or retracting the piston follows and interacts with the shock. In the retraction case a second shock follows the rarefaction wave.

Part I contains a description of the problems treated and the results; for instance, it is found that the strength of the shock interacting with the rarefaction wave asymptotically decreases as $t^{-1/2}$. The second part reviews the approximation procedure (essentially a formulation of the problem which omits terms of third order and higher in the shock intensity) and exhibits the solutions of the above problems. Part III deals in a similar manner with the two-dimensional flow past a pointed obstacle, that is the attached shock problem. Emphasis is given to the asymptotic properties of the flows. G. F. Carrier, USA

341. Anneli Lax, "Decaying shocks," *Commun. appl. Math.*, Sept. 1948, vol. 1, pp. 247–257.

A finite difference technique is used to compute the solution associated with the decay of a shock interacting with a rarefaction

wave. The results are compared with those given by the method of the paper of the foregoing review. The results are in good agreement. G. F. Carrier, USA

342. **Harvard Lomax and Loma Sluder, "Downwash in the vertical and horizontal planes of symmetry behind a triangular wing in supersonic flow,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1803, Jan. 1949, pp. 1–48.

This report is a continuation of a similar study covered by a previous report on the same subject. Detailed formulas and results are presented for the down-wash pattern in vertical and horizontal planes, other than those through the plane of the wing. The report deals exclusively with triangular (delta) wings in supersonic flow, but with subsonic leading edges. Results are given for the chord plane in the extended vortex wake of the wing and for the vertical plane of symmetry up to about 20 per cent of the wing span above the plane of the wing.

The method is based on the distribution of supersonic doublets, and the extensive and detailed mathematical manipulations of this method are shown to serve as a guide for extending the calculations to other planforms. Comparison is also made of the results obtained by the more detailed method with those of an approximate method given in the previous report.

E. Arthur Bonney, USA

343. **C. S. L. Robinson, "Flow of a compressible fluid through a series of identical orifices,"** *J. appl. Mech.*, Dec. 1948, vol. 15, pp. 308–310.

This paper analyzes a method of calculation for the flow of a gas through a number of identical orifices. The computation makes use of the general equation for a single orifice and certain additional assumptions. A number of curves are presented showing the flow factor and over-all critical pressure ratio as function of the number of orifices in series. H. E. Sheets, USA

344. **Seiichi Ōyama, "Application of the hodograph method to the compressible flow with circulation past an elliptic cylinder"** (in Japanese), *Rep. Inst. Sci. Technol. Tokyo*, May-June 1948, vol. 2, pp. 63–66.

The approximate hodograph method of von Kármán and Tsien was generalized to the case of flow with circulation by Tahito Ishida [*Rep. aerotech. Inst. Army Japan*, no. 2376, 1942] and by Isao Imai [*Rep. aero. Res. Inst. Tokyo*, no. 323, 1945]. The author applies the method to the flow with circulation past an elliptic cylinder of thickness ratio 1/10. The distortion of the profile shape in the physical plane is closely investigated, and is shown to become considerable as circulation increases. Calculations are also carried out concerning the accuracy of von Kármán's pressure coefficient formula, and it is concluded that it may be used with sufficient accuracy even when circulation exists.

Humio Tamaki, Japan

345. **John Stack, "Tests of airfoils designed to delay the compressibility burble,"** *Nat. adv. Comm. Aero. Rep.*, no. 763, 1943 (issued in 1947), pp. 1–14.

While this report was written in 1943, and presumably does not represent latest knowledge, it still is of interest. The airfoils tested were designed to have as far as practicable uniform chordwise pressure distribution, and thus minimum local induced velocity.

Experiments have shown that when the translational velocity of an airfoil plus the maximum local induced velocity equals local sonic velocity, rapid adverse changes in lift and drag coefficient occur (critical Mach number). Therefore a decrease in the maximum induced velocity should permit higher translational velocities, and the airfoils were designed on this theory. The mean camber line was determined by an equation based on Glauert's expressions for induced velocity, and the thickness form was determined by trial.

The test data show both increased critical Mach number and reduced drag at low velocities as compared with previous airfoil shapes, but the angle-of-attack range is less. The decreased range may result in stalling at take-off if the new shape is used for propeller blades, but there are probably many applications in which advantage could be taken of the better efficiency and improved characteristics of these airfoils at high velocities.

C. W. Smith, USA

346. **G. F. Carrier, "On the stability of the supersonic flows past a wedge,"** *Quart. appl. Math.*, Jan. 1949, vol. 6, pp. 367–378.

The author investigates the stability of the two kinds of shocks that can be attached to a two-dimensional wedge, with respect to infinitesimal disturbances. It is concluded that both configurations are stable.

The equations for small disturbances are written down, and the velocity field is resolved into one part depending on the nonuniform entropy and another irrotational part. The boundary conditions at the shock are carefully set up.

Certain types of reflection of plane waves from the shock are considered in detail, without including the solid boundary. Including the solid boundary, other types of solutions are found by separation of variables. It is shown that solutions obtained by linear superposition of these simple solutions cannot satisfy the boundary conditions at the vertex of the wedge. It is then conjectured, on the basis of this and other arguments, that the flow configuration is stable.

Finally, it is concluded that the weak shock (supersonic flow downstream) is actually observed because the downstream disturbances cannot reach the neighborhood of the vertex of the wedge, and influence the shock position, while for a strong shock (subsonic flow downstream) disturbances can accumulate near the vertex and eventually cause a transition to the weak shock. J. I. Levinson's work is criticized. C. C. Lin, USA

EDITOR'S NOTE: A note added in proof says: ". . . the analysis of the foregoing problems is not complete . . . we must still investigate how an arbitrary disturbance grows with time. This problem has been solved for certain special cases and a continued treatment of this stability question will appear as Part II of this paper."

347. **Hans Reissner, "Blade systems of circular arrangement in steady, compressible flow,"** *Courant Anniv. Vol.*, Interscience Publishers, New York, 1948, pp. 307–327.

The problem studied by the author is the direct problem of dimensioning a compressor or a turbine. In other words, given the variation of pressure over every blade section and of the axial component of velocity along the axis for axial-type machines (or of the radial component for centrifugal machines), the compression (or expansion) ratio, the form of the airfoils and the field of flow around them are determined.

For infinitely many blades a solution by simple quadratures, starting from Euler's continuity and energy equations, is obtained in the case when the motion has a potential, as well as when there is no potential. Under these assumptions the motion does not depend on the angular co-ordinate and the blade system is replaced by a continuous symmetrical force field.

In order to allow for finite spacing of the blades, which gives rise to nonuniform flow in circumferential direction, the author avails himself of a procedure based on the assumption that the streamline surface halfway between two consecutive blades is

the same as in the case of uniform flow. The corrective terms for the nonsymmetric flow, within the space between two blades, to be added to the flow in the case of infinitely many blades, are expanded in power series in terms of the angular distance ψ from the above-mentioned streamline surfaces.

Thus the author obtains recurrence formulas providing these corrective terms. The solution is still simple if one keeps the terms which are linear in ψ only; this corresponds with the assumption of linear pressure variation from the upper surface of one blade to the lower surface of the consecutive one over every circumference having its center on the axis.

Carlo Ferrari, Italy

348. A. A. Nikolsky and G. I. Taganoff, "Flow of a gas in a local supersonic zone and some conditions for the breakdown of potential flow," *Hdqtrs. Air Mat. Comm. Dayton Transl.*, A9-T-17, 1948, pp. 1–43 (trans. from *Appl. Math. Mech. (Prikl. Mat. Mekh.)*, 1946, vol. 10).

The authors discuss the flow conditions in a local supersonic zone adjacent to an airfoil at high subsonic Mach numbers. For the shock-free case they deduce a series of important theorems, including the following: (*a*) On the transition ($M=1$) line, the inclination of the velocity vector varies monotonically. (*b*) On each characteristic line within the supersonic region the magnitude and inclination of the velocity vector vary monotonically. (*c*) The supersonic zone of the physical plane maps into the hodograph plane with a one-to-one correspondence.

By use of these theorems they show that it is impossible for a Prandtl-Meyer flow to be embedded inside a shock-free flow, and that shock-free flow is impossible if the profile boundary has a straight-line segment adjacent to the supersonic zone. A criterion for the breakdown of the shock-free flow is also developed; however the criterion is not easily applicable to actual flow problems.

H. J. Stewart, USA

349. Clinton E. Brown, "Theoretical lift and drag of thin triangular wings at supersonic speeds," *Nat. adv. Comm. Aero. Rep.*, no. 839, 1946 (issued in 1948), pp. 1–8.

The present paper determines the linearized supersonic flow over a triangular wing with a subsonic leading edge, by means of a doublet distribution in the plane of the wing. First the author determines the effect of line doublets, that is of doublets which are arranged along an arbitrary straight line through the tip of the triangle in such a way that the arising flow field becomes conical. Next these line sources are superimposed to fulfill the boundary conditions.

It is remarkable that the integral equation which arises by this process can be solved immediately by the analogy to an incompressible problem. This leads to simple expressions for the pressure distribution and the lift coefficient. The lift-curve slope is found to be a function of the ratio of the tangent of the apex angle to the tangent of the Mach angle. The computation of the drag coefficient requires the determination of the suction force which occurs at the leading edge of the profile. This is carried out by an analogy to a method known from the linearized incompressible airfoil theory.

Gottfried Guderley, USA

350. Kazuo Kondo, "On the fundamental problem in the wing theory of small disturbances" (in Japanese), *J. Soc. appl. Mech. Japan*, Sept. 1948, vol. 1, pp. 146–153.

The author recapitulates the wing theory of small disturbances starting from the classical wave equation. By applying the Huyghens principle and Lorentz transformation, an expression for the pressure field around the wing is obtained, the effects of camber and thickness being clearly shown. A formula for the oscillating wing is also deduced and is proven to be equivalent to Küssner's result.

Itiro Tani, Japan

351. Clinton E. Brown and Hermon M. Parker, "A method for the calculation of external lift, moment, and pressure drag of slender open-nose bodies of revolution at supersonic speeds," *Nat. adv. Comm. Aero. Rep.*, no. 808, 1945 (issued in 1948), pp. 1–8.

An approximate method is presented for calculating the external lift, moment, and pressure drag of slender open-nose bodies of revolution at supersonic speeds. It is assumed that the duct can accommodate the full mass flow corresponding to its inlet area and the free-stream speed, so that the external flow is independent of the behavior of the internal flow.

The method of perturbations is used, leading to the linearized potential equation which is solved by the method of sources and sinks for the axial flow and doublets for the cross flow. Approximate boundary conditions are satisfied. A section is included giving the calculation procedure for a numerical solution. Lift, moment, and pressure drag are calculated for a particular body at $M = 1.45$, 1.60, 1.70 and 3.00. The lift and moment results compare favorably with available experimental data; drag results are not compared due to the uncertainty of skin-friction drag in the experimental data. The calculations indicate an increase in lift coefficient with increase in Mach number and decreases in pressure-drag and mid-point moment coefficients with increase in Mach number.

Lester L. Cronvich, USA

352. H. Poritsky, "Linearized compressible flow," *Quart. appl. Math.*, Jan. 1949, vol. 6, pp. 389–403.

In another paper [Poritsky, *Quart. appl. Math.*, 1949, vol. 6, pp. 379–388] the author has shown that Laplace's equation has the solution

$$f(Z),\ Z = (x + iy)/(R + z),\ R^2 = x^2 + y^2 + z^2 \quad \ldots\ldots (1)$$

where f is an arbitrary function. The Prandtl-Glauert equation of linearized supersonic flow, namely

$$\varphi_{xx} + \varphi_{yy} + (1 - M_0^2)\varphi_{zz} = 0 \quad \ldots\ldots (2)$$

can be formally reduced to Laplace's equation by the imaginary change of variable $z = i\beta z'$, $\beta^2 = M_0^2 - 1$. With this change of variable, Z above is replaced by $i\beta(x + iy)/(R' + z)$, where $R'^2 = z^2 - \beta^2(x^2 + y^2)$ and, with this modification, $f(Z)$ of Equation (1) satisfies Equation (2), and so also do its real and imaginary parts. Such solutions are clearly associated with Mach cones of the type $z^2 = \beta^2(x^2 + y^2)$. This idea is then applied, by means of a mapping method, to (*a*) conical flow past a slender cone, (*b*) the decrease of lift due to the edge effect of a rectangular wing, (*c*) the same problem for a trapezoidal wing with an oblique edge within the Mach cone. In case (*b*) it is found that the decrease is 50 per cent over the area within the Mach cone.

It is briefly indicated how other solutions of Laplace's equation obtained by the author in the paper cited above can be adapted to nonconical flows.

L. M. Milne-Thomson, England

353. Richard D. Linnell, "Two-dimensional airfoils in hypersonic flows," *J. aero. Sci.*, Jan. 1949, vol. 16, pp. 22–30.

The hypersonic similarity law of Tsien is used as a basis for developing equations for the lift, drag, and moment coefficients of supersonic airfoils. The results obtained are valid when the product of the Mach number and thickness ratio is of the order of 1. Detailed calculations are made for a flat plate, a symmetrical double wedge, a single wedge with flat lower surface, and a circular

are with flat lower surface. The results obtained correlate satisfactorily at low supersonic Mach numbers with the linearized theory, and at very high supersonic Mach numbers satisfactory agreement is obtained with the results of Sanger and others who used the simplifying assumption of infinite Mach number.

The results presented are an important extension of existing information in that the force characteristics at Mach numbers intermediate between these extremes can now be readily calculated. The optimum airfoil from the standpoint of maximum lift-drag ratio is a wedge with flat lower surface and the maximum thickness at mid-chord. John V. Becker, USA

Turbulence, Boundary Layer, etc.

(See also **Revs. 325, 338, 339, 363, 380, 382, 383**)

354. P. Chiarulli and J. C. Freeman, "Stability of the boundary layer," *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-1197-IA, Aug. 1948, pp. 1–104.

Declassification of the German reports relating to the stability of the boundary layer has enabled the authors to give an orderly presentation and critical review of all of the wartime work on this important subject. Although the introductory remarks mention the experimental work of Schubauer and Skramstad, and of Liepmann, and the theoretical work of Lin and Lees on the stability of a boundary layer in a compressible fluid, the report is mainly an exposition of the stability theory of the two-dimensional incompressible flow in a laminar boundary layer over an arbitrary body.

The first two chapters outline in some detail the derivation of the Orr-Sommerfeld equation for the disturbance stream function and the character of its solutions. The methods of Schlichting, Pretsch, and Lin for computing the stability of a given velocity distribution are described.

The third chapter treats the applications of the stability methods to airfoils and to flows with alternating regions of increasing and decreasing pressure.

Chapter IV discusses the stability of the boundary layer with suction.

Chapter V introduces an "evaluation and correction" of the German work by Freeman. To the reviewer the chapter seems more precisely a selection of the most accurate and reliable values. This selection is assisted by an exact solution of the stability of the asymptotic velocity profile for the boundary layer with suction and by an application of Lin's approximate method to the suction distributions used by Schlichting and by Iglisch. For the asymptotic suction case a value of 41,770 is obtained for the critical Reynolds number by the exact method as compared with 39,700 by Lin's approximate method, 55,000 by Pretsch, and 70,000 by Bussmann and Münz.

A bibliography of 61 titles is included.

Hugh L. Dryden, USA

355. J. H. Preston, "The boundary-layer flow over a permeable surface through which suction is applied," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2244, Feb. 1946 (issued in 1948), pp. 1–29.

The equation of motion in the boundary layer along a permeable flat plate with constant (evenly distributed) suction is established on the basis of the solutions of Griffith and Meredith for the infinite plate, and the solution of Blasius for the semi-infinite plate. Approximative calculations for boundary layers along airfoils are obtained by extending the theory of the plate boundary layer.

The results of the calculations show that the required suction flow to prevent laminar separation at high lift coefficients is much smaller with distributed suction than with local suction through a small number of slots. Furthermore, the investigations of Pretsch show that the stability of a laminar boundary layer is considerably increased when distributed suction is applied. Even with small suction velocities an appreciable stabilization effect can be obtained according to the theory. Thus it seems to be possible to offset the disturbing effects of surface waviness on the present laminar wing designs, and to extend the region of laminar boundary layer over the entire airfoil by using a relatively small amount of distributed suction. Experiments carried out by Ackeret and Pfenninger seem to confirm the theoretical results.

For model tests the distributed suction would be very useful as the scale effect could be compensated by reducing the boundary-layer thickness of the model and thus artificially "increasing" the Reynolds number. A major difficulty for the practical application seems to be the problem of obtaining lightweight porous surfaces with enough strength to be used in the modern stressed-skin constructions. E. Haenni, USA

356. Iwao Shibuya, "Theory of the laminar boundary layer for sudden change of flow, and its application. I" (in Japanese), *J. Soc. appl. Mech. Japan*, May 1948, vol. 1, pp. 94–100.

The analysis of the nonsteady boundary-layer flow has been confined to cases of motion started from rest, or of periodic motion. The author considers another problem of transient phenomena, namely, the case when the velocity outside the layer changes suddenly from a steady value U_0 to $U_0 + U_1$, where U_1 is an arbitrarily given function of time. Assuming U_1 to be small compared to U_0, the differential equation is linearized so as to be solved by the operational method. To simplify the calculations, moreover, the steady velocity is neglected near the wall, while it is assumed equal to U_0 near the outer edge of the layer.

A numerical discussion is given for the case when both U_0 and U_1 are linear functions of the distance along the wall.

Itiro Tani, Japan

357. L. Howarth, "Concerning the effect of compressibility on laminar boundary layers and their separation," *Proc. roy. Soc. Lond. Ser. A*, July 28, 1948, vol. 194, pp. 16–42.

The exact theory of the compressible laminar boundary layer is developed under the two basic assumptions that the viscosity is proportional to the absolute temperature and the Prandtl number is unity. The equation of motion for this case is reduced to a simplified form very similar to the ordinary incompressible equation by making a transformation of the ordinate normal to the layer. The only modification of the incompressible equation is the appearance of a variable multiplicative factor G in the pressure term.

The flow along a flat plate and the flow in jets and wakes are discussed; here the factor G does not enter, due to the zero pressure gradient, and thus the compressible problem is similar to the incompressible one. In all other problems, the divergences (from the incompressible case) produced by the factor G and the change of scale must both be discussed. Two examples are considered in detail, for which the velocity distributions at several Mach numbers were computed with the aid of electronic machines. Solutions are obtained for the accelerated flow along a flat plate where the velocity in the main stream increases linearly from a stagnation point, and for retarded flow with linearly decreasing velocity in the main stream.

The general implications of the theory are discussed and qualitative conclusions drawn when the main-stream velocity starts from a stagnation point, rises to a maximum and subsequently falls. It is concluded that for such a velocity distribution increas-

ing compressibility will reduce the skin friction, increase the boundary-layer thickness, and cause earlier separation as compared with the incompressible flow with the same main-stream velocity distribution and the kinematic viscosity corresponding to conditions at the stagnation point. J. S. Isenberg, USA

358. Eberhard Hopf, "A mathematical example displaying features of turbulence," *Commun. appl. Math.*, Dec. 1948, vol. 1, pp. 303–322.

Various attempts have been made in the past to construct mathematical solutions of systems of equations, related to the Stokes-Navier equations, in order to illustrate in a simplified way the important features of turbulent flow. The present paper gives a new example of this approach. Unfortunately the Fourier components of the solutions of the equations adopted, belonging to different frequencies, behave completely independently of one another. This is in marked contrast to the behavior of real fluids, according to current theories of turbulence.

H. B. Squire, England

Aerodynamics of Flight; Wind Forces

(See also Revs. 298, 299, 326, 331, 333, 335, 336, 342, 345, 349, 350, 351, 374, 377)

359. J. Whatham and H. M. Lyon, "A theoretical investigation of dynamic stability with free elevators," *Rep. Memo. aero. Res. Counc. Lond.*, no. 1980, Mar. 1943 (issued in 1948), pp. 1–19.

It is shown that an unstable pitching oscillation similar to directional snaking can occur with a free elevator if it is closely balanced aerodynamically and has no mass balance, or if an inertia weight is added. In the critical region close to the stability boundary a small increase in moment of inertia or decrease in b_2 (moment coefficient about elevator axis per unit of elevator angle deflection) may change a well-damped oscillation into a rapidly growing one. On the stable side of this critical region, elevator moment of inertia and damping have very little effect on the motion, and can be neglected in investigations of maneuverability and response to gusts. The theoretical analysis is based on Routh's stability criteria for the equations of motion. While the complete equations are of the sixth order, by neglecting either the change in velocity, or the elevator inertia and damping, a fourth-order equation is obtained. Numerical calculations for the Spitfire are included. Andrew Vazsonyi, USA

360. G. Santangelo, "Static and dynamic longitudinal stability of an airplane with free and locked control (Stabilità longitudinale statica e dinamica dei motovelivoli a comando bloccato e libero)," *Monogr. sci. Aero.*, Feb. 1947, no. 4, pp. 3–60.

The theory of the static and dynamic stability of an airplane is reviewed for both stick-fixed and stick-free cases, with particular reference to the effects of slip stream and propellers. Compressibility and aeroelastic effects are not taken into account. Much of the subject matter can be regarded as classical, but some general empirical curves and figures deduced from wind-tunnel test data are of interest. The notation differs considerably from that used in the United States and Britain.

A. D. Young, England

361. Harold B. Pierce, "Gust-tunnel investigation of a 45-deg sweptforward-wing model," *Nat. adv. Comm. Aero. tech. Note*, no. 1717, Oct. 1948, pp. 1–14.

Results of tests on a 45-deg sweptforward-wing model verified values of acceleration increment calculated by using: (1) the slope of the lift curve of an equivalent straight wing multiplied by the cosine of the angle of sweep, and (2) strip theory to determine the effect of gradual penetration of the gust by the swept wing. Similar tests and calculations are given in *Nat. adv. Comm. Aero. tech. Note*, no. 1528, for a 45-deg sweptback-wing model.

John Kirkland, USA

362. P. E. Montagnon, A. Robinson, and S. V. Fagg, "A note on the interpretation of V-g records," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2097, Jan. 1945 (issued in 1948), pp. 1–8.

The occurrence of a reading of large acceleration on a V-g slide tends to mask the occurrence of smaller accelerations at the same speed. If, however, it is assumed that the distributions of accelerations in a set of slides are not correlated, a formula is obtained for the number of "nonmasked" readings of the smaller values. By starting at the high-acceleration end of the distribution curve, where the error should be negligible, one can proceed downward using this formula to determine the true number of occurrences at each value.

To check this, a flight-analyzer record covering 305 hours of B-17 operational flying was employed. Such a record, as distinguished from V-g slides, gives a complete plot of acceleration and air speed against time. First the nonmasked readings that would have shown up on a slide were counted. The procedure described above was then used to estimate the true occurrence frequencies. The agreement with the frequencies actually observed is good.

W. R. Sears, USA

363. A. S. Batson and J. H. Preston, "The effect of boundary-layer thickness on the normal force distribution of aerofoils, with particular reference to control problems," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2008, Apr. 1942 (issued in 1948), pp. 1–46.

This report combines four short papers previously issued in 1942 as British Aeronautical Research council reports. The investigation consisted of pressure-distribution, wake-traverse and smoke-flow tests of two airfoils of 10 and 20 per cent thickness ratio in a 7-ft low-speed wind tunnel at the National Physics Laboratory. The objective of the work was to establish the reasons for the marked effects of boundary-layer thickness on control characteristics. The airfoil models were not actually fitted with movable controls, but the pressures were integrated about points at 20, 30, and 40 per cent chord ahead of the trailing edge to evaluate the hinge moments of hypothetical unbalanced controls. Wires were used to vary the transition point and boundary-layer thickness. The test measurements are presented in detail and comparison is made with theory to aid in determining the magnitude and character of the boundary-layer effects. Analysis of the results indicated that the lift-curve slope and hinge-moment parameters for a given control chord could be correlated by a single curve which was approximately independent of airfoil thickness, Reynolds number and boundary-layer thickness. The authors call attention to the fact that model tests in relatively turbulent wind tunnels will result in appreciably smaller values of the hinge-moment parameters $dc_h/d\alpha$ and $dc_h/d\delta$ than will obtain in flight. John V. Becker, USA

364. Thomas D. Reisert, "Gust-tunnel investigation of a wing model with semichord line swept back 30 deg," *Nat. adv. Comm. Aero. tech. Note*, no. 1794, Jan. 1949, pp. 1–14.

This is the third of a series of reports providing data for the prediction of gust loads that may be anticipated on airplanes incorporating swept wings.

The results, although limited, indicate that the maximum gust load depends on the lift-curve slope of an equivalent straight wing

multiplied by the cosine of the angle of sweep, and on the effect on the unsteady-lift function of gradual penetration of a sweptback wing into a gust. John R. Spreiter, USA

365. **A. E. Knowler and F. W. Pruden, "The effect of brake flaps on an aerofoil at high speeds,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2211, June 1942 (issued in 1948), pp. 1–23.

This report presents measurements on the effect of brake flaps on an airfoil at high speeds. The measurements were made in the circular High-Speed Tunnel by the National Physical Laboratory for application to a low-winged two-engined aircraft.

The flaps tested varied in size, shape and position relative to the airfoil, and results were obtained over a Mach number range of 0.22 to 0.85. The pressure distribution in the airfoil wake in the neighborhood of the tail plane was also found and included in this report. B. H. Edelman, USA

366. **Jack Fischel, "Wind-tunnel investigation of an NACA 65-210 semispan wing equipped with circular plug ailerons and a full-span slotted flap,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1802, Jan. 1949, pp. 1–62.

Results of a wind-tunnel investigation are presented for three configurations of circular-plug (spoiler) ailerons on an unswept wing with full-span flaps for speeds up to $M = 0.61$. While no unusual results were encountered, one configuration exhibited characteristics somewhat superior to those usually found for this type of control surface. J. M. Wild, USA

367. **Victor I. Stevens, "Theoretical basic span loading characteristics of wings with arbitrary sweep, aspect ratio, and taper ratio,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1772, Dec. 1948, pp. 1–33.

This report presents a rapid method of calculating the basic span loading and associated aerodynamic characteristics of wings with arbitrary sweep, aspect ratio, taper ratio, and twist. The method is based on the Weissinger lifting line method and shows accuracies comparable to that of lifting surface methods. The method is capable of predicting basic loading characteristics, twist required for control of separation, and of evaluating the effects of aeroelastic deformation. Corrections for compressibility effects are also given. The present report supplements an earlier report by John De Young ["Theoretical additional span loading characteristics of wings with arbitrary sweep, aspect ratio, and taper ratio," *Nat. adv. Comm. Aero. tech. Note*, no. 1491, 1947]. These two reports permit rapid evaluation of basic and additional types of loading and associated characteristics of wings having sweep angles from −45 deg to 75 deg, aspect ratios of 1.5 to 10, and taper ratios of 0 to 1.5. H. P. Liepman, USA

368. **Abbott A. Putnam, "An analysis of life expectancy of airplane wings in normal cruising flight,"** *Nat. adv. Comm. Aero. Rep.*, no. 805, 1945 (issued in 1948), pp. 1–9.

The purpose of this report is to provide a basis for judging the relative importance of wing failure by fatigue and by single intense gusts. The analysis, which is based upon previously published data on the frequency of atmospheric gusts, contains the effect of stress-concentration factor, stress-load relation, wing loading, design and cruising speeds, design gust velocity and airplane size.

The results of the analysis indicate that the detail design and construction influence the fatigue life but do not appreciably affect the single-gust life, while exactly the opposite can be said for the effect of operating speed and changes in design gust velocity. For conventional design and normal operating conditions, the fatigue life and single-gust life are of about equal importance, and neither are much affected by the trends in design toward higher wing loading, reduced-load factor, larger size and increased speed. Harvard Lomax, USA

369. **A. M. Peiser and M. Wilkerson, "A method of analysis of V-G records from transport operations,"** *Nat. adv. Comm. Aero. Rep.*, no. 807, 1945 (issued in 1948), pp. 1–7.

The problem of life expectancy of aircraft calls for a method permitting the designer to predict the number and magnitude of large accelerations which would be experienced in a given number of hours of normal transport operation. This report describes such a method, involving analysis of V-g (speed-acceleration) data from any transport airplane. This method makes use of standard statistical procedures (involving Pearson Type III probability curves) to derive "flight envelopes" of V-g records. It is possible to predict that, on the average, in a given number of flight hours, one value of air speed, and one positive and one negative acceleration increment will exceed the envelope, with equal probability for all air speeds. Comparisons of the flight envelopes with data indicate a high degree of accuracy of the method in predicting large accelerations and air speeds.

Harvard Lomax, USA

370. **Joseph Weil and William C. Sleeman, Jr., "Prediction of the effects of propeller operation on the static longitudinal stability of single-engine tractor monoplanes with flaps retracted,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1722, Oct. 1948, pp. 1–42.

The effects of propeller operation on the static longitudinal stability of single-engine tractor monoplanes are analyzed, and a simple method is presented for computing power-on pitching-moment curves for flap-retracted flight conditions. The methods evolved are based on the results of powered-model wind-tunnel investigations of 28 model configurations. Correlation curves are presented from which the effects of power on the down-wash over the tail and the stabilizer effectiveness can be rapidly predicted. The procedures developed enable prediction of power-on longitudinal stability characteristics that are generally in very good agreement with experiment. Andrew Vazsonyi, USA

371. **H. Reissner and M. Morduchow, "A theoretical study of the dynamic properties of helicopter-blade systems,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1430, Nov. 1948, pp. 1–114.

In this paper the authors treat the problem of stability of an individual blade of a helicopter rotor under certain simplifying assumptions. In the derivation of the aerodynamic forces and moments the assumption is apparently made that these are always related to the instantaneous effective angle of attack in the same way as in a steady-state condition of flight. The same assumption is made in stability calculations for conventional aircraft, but it is probably of doubtful validity when applied to a rapidly oscillating helicopter blade. Another assumption that is made is that the blade is perfectly rigid and that all motion takes place about a spherical bearing near the hub of the rotor.

With these restrictive assumptions the authors go on to calculate the modes of vibration of a freely vibrating helicopter blade, both in the state of hovering and in a state of low-speed horizontal travel. In the case of hovering the vibration will be like an ordinary case of damped free vibration, and four such cases were investigated. In the case of low-speed travel the blade is acted on by aerodynamic shaking forces which are functions of the angular position of the blade but of relatively small magnitude due to the low speed. If the blade is now set vibrating, the above-men-

tioned shaking forces will also depend on these free vibrations and will in turn affect them; hence, they will be different from the free vibrations in hovering. This difference was calculated by an approximate method which yields good results for low traveling speeds, and it was found that each mode now consisted of the sum of several vibrations of different frequencies and related amplitudes; the vibration with the largest amplitude in each mode corresponded to the mode in hovering.

A numerical example is worked out for each case; this makes the results very interesting. N. O. Myklestad, USA

372. Warren A. Tucker, "Characteristics of thin triangular wings with triangular-tip control surfaces at supersonic speeds with Mach lines behind the leading edge," *Nat. adv. Comm. Aero. tech. Note*, no. 1600, May 1948, pp. 1–33.

On the basis of earlier studies, principally that of A. E. Puckett ["Supersonic wave drag of thin airfoils," *J. aero. Sci.*, 1946, vol. 13, no. 9], the author has investigated the control characteristics of thin triangular wings with triangular-tip control surfaces. The planform of the control surfaces is geometrically similar to that of the wing as a whole. The treatment is for supersonic flight at Mach numbers high enough to cause the Mach line from the tip to lie behind the leading edge. In line with the underlying work of Puckett, the several assumptions and approximations of the linearized theory are embodied in the treatment.

Analyses are made of lift effectiveness, when both control surfaces are deflected in the same direction, the pitching-moment variation resulting from flap deflection, the rolling-moment effectiveness when the control surfaces are deflected in opposite directions, the hinge moment accompanying control-surface deflection and the hinge moment due to the angle of attack of the entire wing.

If one compares the delta wing with a two-dimensional wing as to control-surface efficiency, which is a measure of the lift increment produced by a given control force, it is found that for equal ratios of flap area to wing area, the two situations are identical. In regard to rate of roll, however, for given control force, the triangular wing will roll twice as fast for equal ratios of flap area to wing area.

Another conclusion was that the hinge-moment coefficient due to angle of attack was large compared either to the hinge-moment coefficient due to control-surface deflection or to the two-dimensional hinge-moment coefficient due to angle of attack. There is thus a balancing effect which tends to reduce the stick force as soon as the attitude of the wing has changed, as in a steady roll. This effect would have to be considered in connection with stick-free longitudinal stability if this triangular wing-flap combination were used as a horizontal tail.

Pressure forces alone are considered, viscosity effects being neglected. Stewart Way, USA

Aeroelasticity (Flutter, Divergence, etc.)

(See also Revs. 313, 324, 336, 350, 367, 371)

373. William H. Phillips, "Theoretical analysis of oscillations of a towed cable," *Nat. adv. Comm. Aero. tech. Note*, no. 1796, Jan. 1949, pp. 1–19.

Assuming a uniform cable under constant tension, the differential equation for transverse oscillations is found to have the form

$$\frac{\partial^2 y}{\partial x^2} - C_1 \frac{\partial y}{\partial x} - C_2 \frac{\partial y}{\partial t} = \frac{1}{a^2} \frac{\partial^2 y}{\partial t^2}.$$

This is the well-known wave equation modified by the addition of the second and third terms which approximate the effect of the aerodynamic forces acting on the tow cable. Based on a number of simplifying assumptions, expressions are developed for the parameters C_1 and C_2. Using the above equation, the solution for steady-state oscillations of an infinitely long cable is developed.

It is found that oscillations traveling downwind along the cable are amplified by aerodynamic forces when the air speed is greater than the speed of propagation of waves along the cable. Waves traveling upwind along the cable are always damped. The application of the theoretical analysis to the actual behavior of a cable towing an air-speed head is discussed. It is concluded that this theory provides a possible explanation for the violent motions of towed bodies which appear above a certain speed.

Dana Young, USA

374. Allen R. Stokke and William S. Aiken, Jr., "Flight measurements of buffeting tail loads," *Nat. adv. Comm. Aero. tech. Note*, no. 1719, Oct. 1948, pp. 1–19.

This report presents the results of a systematic and comprehensive flight-test investigation of the magnitude and nature of the horizontal tail loads encountered on a fighter-type airplane when operated up to the buffet boundary. At Mach numbers below 0.64, buffet occurred simultaneously with the attainment of the airplane's maximum normal-force coefficient, but at higher Mach numbers, when compressibility effects predominated, the buffeting began earlier.

The magnitude of the buffet increment of tail load decreased with increasing Mach number to the maximum investigated of 0.8 at constant dynamic pressure. The total tail load, the sum of the maneuvering tail load and the buffet increment, is critical at low speed and high angle of attack for an aft center of gravity. An interesting by-product of the investigation is the confirmation of the fact that the maximum lift coefficient attained is a function of the rate of angle-of-attack increase.

The results have been carefully analyzed. Comparison of the natural frequencies of the various airplane components with measured buffet frequencies is made. The boundaries of the buffet regions in normal-force coefficient versus Mach number charts are shown. The magnitude of the relieving effect in the tail loads due to inertia is computed from the measured linear acceleration on the tail. Conrad A. Lau, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 291, 347, 370, 371)

375. John D. Stanitz, "Two-dimensional compressible flow in conical mixed-flow compressors," *Nat. adv. Comm. Aero. tech. Note*, no. 1744, Nov. 1948, pp. 1–75.

This paper discusses the ideal compressible flow through runners in which the center line of the passage generates a right circular cone about the axis of the runner. With the assumption of two-dimensional isentropic flow, the conditions depend on co-ordinates consisting of the conic radius R (distance along conic element from apex of cone) and an angle θ. The method is valid only for sufficiently large cone angles. For the numerical solution of the equations of motion by the relaxation method, the R, θ co-ordinates are transformed so that the arbitrary blade shapes become straight and parallel; this transformation is established by making the velocity-potential lines and streamlines of the corresponding incompressible flow the Cartesian co-ordinates in the transformed plane.

The boundary conditions are: On the trailing side of one of the blades the value of the stream function is given by the discharge of the runner and the number of its blades, while its value on the

driving side of the adjacent blade is zero. The Kutta condition of flow tangency must be satisfied at least at the outer tips of the blades. Methods of estimating these stream-function values are discussed in the paper. One of them deals with what the author calls "impeller slip factor." This is a figure from which the actual effect of a finite number of blades can be compared with that of an infinite number of blades determined by an elementary (one-dimensional) theory of turbomachines (Euler's theory).

The author gives a completely calculated example of the simplest case, namely a radial discharge compressor with straight radial blades and variable passage height (cone angle 180 deg). Diagrams of streamlines, lines of constant pressure and of constant Mach number are given. They show very interesting details which remind the reviewer of his own results obtained in 1925, which gave the first exact solution for the potential incompressible flow through a radial discharge runner with constant passage height [*Z. angew. Math. Mech.*, 1925, vol. 5, p. 481; and *Nat. adv. Comm. Aero. Doc.*, no. 6410/791]. One of the most interesting of these details is the formation of a wheel-type eddy on the driving face of the blade. This phenomenon, which can be explained in the main by approximate methods (one of them is given by the author in his paper), is a very important one since its occurrence would make the actual flow differ essentially from the ideal one. One of the means to avoid this disturbance, not mentioned by the author, is to increase the number of the blades.

Whether the generally very lengthy processes giving exact solutions of the ideal flow are worth while for practical purposes remains to be decided by the designers of turbomachines.

Wilhelm Spannhake, USA

376. **M. J. Lighthill, "A mathematical method of cascade design,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2104, June 1945 (issued in 1947), pp. 1–18.

In an earlier paper [Lighthill, "A new method of two-dimensional aero-dynamic design," *Rep. Memo. aero. Res. Counc. Lond.*, no. 2112, 1945] the author provided a method for designing an airfoil with a given velocity distribution. The present paper is an extension of this method to cascade design by transforming the cascade into a single airfoil. An example is worked out in detail and suggestions are made to aid in the selection of appropriate values of the parameters. J. M. Wild, USA

377. **Jerome B. Hammack, "Flight investigation in climb and at high speed of a two-blade and a three-blade propeller,"** *Nat. adv. Comm. Aero. tech. Note*, no. 1784, Jan. 1949, pp. 1–42.

As part of an NACA program of propeller tests, two- and three-blade propellers were tested on a slender-nose fighter airplane equipped with slip-stream survey apparatus. The test results show some effects of blade loading and of tip Mach numbers up to 1.03. Compressibility losses were encountered at all tip Mach numbers above 0.90, and here the shank sections were found to be particularly bad. Forward Mach number did not exceed 0.71.

W. R. Sears, USA

Flow and Flight Test Techniques

(See also Revs. 339, 355, 374)

378. **S. S. Penner and R. C. Whiteman, "Impulse determinations of rockets by means of rotating systems,"** *Rev. sci. Instrum.*, July 1948, vol. 19, pp. 428–435.

In this paper the author discusses two simple mechanical rotating systems which have been used to measure the specific impulse of rocket propellants. The first system is a freely rotating horizontal wheel upon the periphery of which is mounted a small rocket motor containing the propellant. The change in angular velocity of the wheel caused by the thrust of the rocket is recorded photographically and the impulse as a function of time is calculated from this record. The rotating bar works on the same principle, additional data on chamber pressure as a function of time being obtained by the use of strain gages.

Henry J. Barten, USA

379. **C. N. H. Lock and J. A. Beavan, "Tunnel interference at compressibility speeds using the flexible walls of the rectangular high-speed tunnel,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2005, Sept. 1944 (issued in 1948), pp. 1–37.

A method is presented for obtaining free-air two-dimensional airfoil data at high subsonic Mach numbers in a closed tunnel. This is accomplished by adjusting the flexible walls above and below the airfoil in such a manner as to produce free-air streamlines at the walls. The success of such a method depends, of course, on how accurately these streamlines can be reproduced.

The theoretical aspects of the problem are considered and lead to an approximate method of setting the walls. This consists of setting the walls halfway between the straight and the position for uniform pressures. A modification to this procedure based on comparison of experimental data with theoretical results is presented. The cases of finite lift and wake blockage are considered.

The results presented are a considerable help in assessing the degree to which tunnel results can be trusted at high subsonic speeds. Tunnel results as obtained by this method are valid until the shock developing on the airfoil intercepts the tunnel wall. Henry H. Hoadley, USA

380. **R. W. F. Gould, "A circuit for compensating hot wires used in the measurement of turbulence,"** *Rep. Memo. aero. Res. Counc. Lond.*, no. 2240, Aug. 1945 (issued in 1947), pp. 1–7.

An improved compensating circuit for hot-wire anemometers is described. It consists of a resistance-capacity network in the cathode circuit of a pentode.

The amplified signal generates a feedback voltage across this network in a direction to reduce the input voltage between grid and cathode and of an amount which decreases with increasing frequency. Hence the frequency characteristic can be made to conform to the requirements of a compensation stage.

Hugh L. Dryden, USA

381. **J. F. Manildi, "Testing of jet engines under simulated air-speed and altitude conditions,"** *Aero. Engng. Rev.*, Feb. 1949, vol. 8, pp. 34–38.

A preliminary discussion is given of problems that arise in testing the performance of jet engines. The use of pressurized and evacuated tanks communicating respectively with the inlet and the exhaust is suggested. No experimental data are presented.

Bruce L. Hicks, USA

382. **R. Betchov, "Spectral analysis of turbulence (L'analyse spectrale de la turbulence),"** *Proc. kon. Ned. Akad. Wet.*, Nov. 1948, vol. 51, pp. 1063–1072.

This paper consists of (1) a brief review of the general principles of the measurement of a power spectrum with band-pass filters; (2) a circuit analysis of the particular type of filter actually used; (3) presentation of the measured spectrum of the free-stream turbulence in a small wind tunnel.

In Part (1) the author deduces the integral expression for the mean square voltage output from a given filter fed by a signal

with a given spectrum. He then neglects the effect of filter shape—except for defining the effective band width—and so obtains the simple algebraic relation which is strictly true only for a rectangular band of extremely small width.

Part (2), a network analysis for the frequency response of the Wien-bridge type of filter, should be quite useful for prospective experimenters.

The measured (one-dimensional) power spectrum of turbulence presented in Part (3) seems to show an unexplained anomalous behavior in that the curve drops rapidly for frequencies below 40 cycles. The author states that acoustic energy is negligible, that background noise has been eliminated, and that appropriate correction has been made for the reduced amplifier gain in this low-frequency range [see R. Betchov and E. Kuyper, "Un amplificateur pour l'étude de la turbulence d'un écoulement d'air," *Proc. kon. Ned. Akad. Wet.*, 1947, vol. 50, no. 9].

Stanley Corrsin, USA

383. A. A. Hall, "Measurements of the intensity and scale of turbulence," *Rep. Memo. aero. Res. Counc. Lond.*, no. 1842, Aug. 1938 (issued in 1947), pp. 1–38.

This report gives the results of some experimental work carried out in the Cambridge Aeronautics Laboratory before the second world war. In spite of being about 10 years old the report still represents a basic work in the field of turbulence investigation. All three turbulent components have been measured, the one in the mean stream direction by the hot-wire anemometer method, and the two others by the diffusion of heat as discussed by G. I. Taylor. The readings are taken on the mirror galvanometer and dynamometer. The complete hot-wire equipment has been described, giving diagrams for the bridge and amplifier as well as the calibrating unit.

Measurements were made of the root-mean-square variation at points on a radial traverse of a circular pipe 1.5 in. diam at two values of the Reynolds number. Also the correlation coefficient in the pipe has been determined and the agreement with previous results is good, giving a check on the correct operation of the measuring equipment.

In order to produce a low-turbulence wind tunnel, experiments have been made in a closed return tunnel. The effect of contraction on wind-tunnel turbulence was investigated, and turbulence measurements were made with and without honeycomb. In the low-turbulence wind tunnel, measurements were made of the decay of turbulence produced by square meshes of different mesh sizes, and the Reynolds number effect was examined. Finally the results are compared with measurements previously made at the NPL and by the NACA. P. N. Brandt-Møller, Denmark

Thermodynamics

(See also Revs. 320, 389, 409)

384. R. S. Silver, "Temperature and pressure phenomena in the flow of saturated liquids," *Proc. roy. Soc. Lond. Ser. A*, Nov. 9, 1948, vol. 194, pp. 464–480.

The flow of a saturated liquid through a cylindrical orifice is analyzed on the basis of the following assumptions: A central core of liquid is surrounded by vapor which evaporates from the surface of the central core and which fills the space between the core and the passage walls. The vapor is normal saturated vapor in equilibrium with the liquid surface. The vapor and liquid at any pressure move with equal velocities. The temperature change, extending from the inlet temperature to the saturation temperature at each pressure along the length of the stream, is concentrated in a "cooled annulus" adjacent to the surface of the liquid core. Its gradient is a constant at any pressure. The heat transfer outward through the cooled annulus is equal to the enthalpy of vaporization of the liquid evaporated from the surface of the core.

The reviewer would like to point out that the distinction between this and the usual assumption of isentropic equilibrium two-phase expansion through an orifice lies in the assumption that (1) virtually all the liquid is at higher temperature than the surrounding vapor, and (2) the amount of vaporization is based upon the equality between the rate of heat transfer through the "cooled annulus," as computed by rather rudimentary methods, and the latent heat of the vaporized fluid.

In case of equality between the surface temperature of the core and the saturation temperature, the calculated flow passes through a maximum when the pressure in the exit plane decreases. Curves are shown for the "critical pressure" and "critical propagation velocities" as functions of length and diameter of the nozzle.

Experiments were made with exhaust pressure at one atmosphere. Experimental flows for 3.4 atm initial pressure are compared with calculated flows for a series of nozzle lengths. Agreement is remarkably good. An attempt to measure the critical pressure by drilling a static pressure hole in the nozzle wall resulted in a marked decrease in flow, but showed a pressure in excess of the exhaust pressure.

A comparison is shown of "theoretical" and "experimental" values of the critical pressure. In the opinion of the reviewer, this is in reality a comparison of the calculated maximum flow with the measured flow. Joseph H. Keenan, USA

385. E. A. Watson, "Fuel systems for the aero-gas turbine," *Proc. Instn. mech. Engrs.*, 1948, vol. 158, no. 2, pp. 187–208.

This paper gives a comprehensive discussion of factors and problems involved in the development of fuel systems of aircraft gas turbines. There are two sections to the paper, the first dealing with the supply of fuel and the second with the atomization of fuel.

The first section discusses the various factors influencing the supply and control of fuel. Several of the factors considered are: influence of pressure and temperature on fuel required; the effect of aircraft speed; the effect of combustion efficiency; factors involved in fuel-control systems.

With regard to the atomization of fuel, the following items are discussed: types of atomizers, laws of discharge, contour of fuel spray, influence of losses, factors influencing the size and distribution of droplets, influence of different rates of flow.

Frank L. Wattendorf, USA

Heat Transfer; Diffusion

(See also Rev. 400)

386. D. Rosenthal and R. H. Cameron, "Temperature distribution in cylinder heated by point source moving along its axis," *Trans. Amer. Soc. mech. Engrs.*, Nov. 1947, vol. 69, pp. 961–968.

This paper presents a solution of the problem of determining the temperature distribution caused by a point source of heat moving at constant velocity along the axis of an insulated solid cylinder of circular cross section. It is postulated that the cylinder is sufficiently long, and also that the point source has been in motion a sufficiently long time so that, to an observer moving with the point source, the temperature distribution is not a function of time. Thus the temperature rise at any point is expressible as a function of radial and axial distance from the moving source. The work is an extension of work previously presented in a paper

[D. Rosenthal, "The theory of moving sources of heat and its applications to metal treatments," *Trans. Amer. Soc. mech. Engrs.*, 1946, vol. 68, pp. 849–866] in which solutions for the temperature distributions caused by moving heat sources in flat plates were given.

Fred E. Romie, USA

387. J. K. Hardy, "An analysis of the dissipation of heat in conditions of icing from a section of the wing of the C-46 airplane," *Nat. adv. Comm. Aero. Rep.*, no. 831, 1945 (issued in 1948), pp. 1–10.

This is a reissue of a classified document. The report discusses the rates of evaporation and heat transfer from a heated wing surface during icing conditions. A discussion of the effects of kinetic heating and moisture interception based upon data available then (1945) is given. Later data [M. Tribus and J. Tessman, "Report on the development of heated wings," *Hdqtrs. Air Mat. Comm. Dayton tech. Rep.*, no. F-TR-4972-ND, Jan. 1946; and Carr B. Neel, Jr., N. R. Bergrun, David Jukoff, and Bernard A. Schlaff, "The calculation of the heat required for wing thermal ice prevention in specified icing conditions," *Nat. adv. Comm. Aero. tech. Note*, no. 1472, Dec. 1947] have modified two statements given in the paper: (1) the droplet sizes and moisture contents normally found in the atmosphere are too large to be appreciably evaporated due to kinetic heating, and (2) there does not appear to be any blowing off of the excess moisture running over the wing. These two changes in proposed mechanism do not detract from the value of this fundamental contribution to the understanding of heated wing anti-icing.

Myron Tribus, USA

388. W. Elenbaas, "Dissipation of heat by free convection," *Philips Res. Rep.*, 1948, vol. 3: Oct., pp. 338–360; Dec., pp. 450–465.

The paper presents an excellent review of work done by the author and others in analyzing the process of heat transfer from a surface to a fluid moving due to natural convection. The five dimensionless numbers (Nusselt, Grashof, Reynolds, Prandtl and Péclet; abbreviated by the first two letters in boldface) obtained from a dimensional analysis of the problem are justified on the basis of the equations of motion, continuity, energy and the rate equations for heat transfer. It is shown that $\mathbf{Nu} = f_1(\mathbf{Gr}, \mathbf{Pr})$ in the case of natural cooling. Neglecting inertia forces, this relation becomes $\mathbf{Nu} = f_3(\mathbf{Gr}\cdot\mathbf{Pr})$.

A comparison is made of the results obtained for the function f_3 from experiment, with those obtained by direct integration of the equations of fluid mechanics and heat transmission for the following cases of heat transfer between a solid surface and fluid flowing in natural convection: flat vertical plate, horizontal cylinder, sphere, vertical cylinder, closely spaced vertical plates (cooling ribs), inner surface of vertical tubes, and horizontal flat plate.

Warren M. Rohsenow, USA

389. H. Schuh, "Some problems of flow, heat transfer and diffusion at laminar flow along a plane plate (Einige Probleme der Strömung, des Wärmeüberganges und der Diffusion bei Laminarströmung längs einer ebenen Platte)," *Öst. Ingen.-Arch.*, Sept. 1948, vol. 2, no. 5, pp. 346–360.

Following the method of E. Pohlhausen ["Der Wärmeautausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung," *Z. Angew. Math. Mech*, 1921, vol. 1, p. 115] for solving the laminar boundary-layer equations for a flat plate in incompressible flow (constant fluid properties), the author extends the method to the case of compressible flow properties (variable with temperature). A set of two integral equations, one for the velocity field and one for the temperature field, is derived for variable properties, assuming a constant specific heat and neglecting frictional heating. An iteration method of solution based upon these equations is given.

Two examples of solution by the above method are presented: one for a highly viscous fluid ($Pr > 10$), and one for air ($Pr = 0.7$).

(1) For the case of a highly viscous fluid it is assumed that only the viscosity varies with temperature and the other properties remain constant. Solutions are given for values $Pr = 12.5$ and $Pr = 100$ as well as for heated and cooled plate conditions. It is shown that the incompressible formulas can be used as a good approximation for the calculation of drag if the properties are referred to the temperature at the outside of the boundary layer, and for the calculation of the heat-transfer coefficient if the properties are referred to the plate temperature.

(2) Solutions are made for the case of air ($Pr = 0.7$) assuming that all the fluid properties vary with temperature. Considering the case where the velocities are not too great, the frictional heat term is neglected. Calculations were carried out for the heated plate for three values of plate temperature. It is shown that the incompressible formulas can be used as a fair approximation for calculating the drag and heat-transfer coefficients if the properties are all referred to either the temperature at the outside of the boundary layer or the plate temperature.

The author gives formulas for the solution of the laminar boundary-layer equations for compressible flow including the frictional heating which would apply to higher velocities. However, solutions are not given, since, as the author indicates, the problem has been investigated by others. For example, see L. Crocco's treatment ["Sullo strato limite laminare nei gas lungo una pareta," *R. C. Circ. Mat. Palermo*, 1940/41, vol. 63].

The task of solving the several integral equations by the author's method for this more general case would appear to be much more arduous than the previous case where frictional heating could be neglected.

The methods of solution developed here are applied to the case of diffusing substances in the fluid flow along the plate whereby a concentration field is calculated in addition to the velocity and temperature fields. The case is treated where finite normal velocities appear at the wall, such as occur with evaporation and condensation of the fluid at the wall.

For copies of a translation of this paper address the Air Documents Division, Intelligence Dept. (MCID), Wright-Patterson Air Force Base, Dayton, Ohio.

Lyle L. Moore, USA

390. C. F. Kayan, "Heat-transfer temperature patterns of a multicomponent structure by comparative methods," *Trans. Amer. Soc. mech. Engrs.*, Jan. 1949, vol. 71, pp. 9–16.

Calculations are made for a thick two-dimensional corner composed of two different materials of unequal thickness, with fluid-boundary conductance on each side, and with temperature difference established between the two boundary fluids. The problem is studied by various methods including the experimental "geometrical" and the network electrical analogies, as well as the solution of the equivalent electrical network by the process of arithmetical iteration. The comparative results are shown.

Alexander Mendelson, USA

391. Gregory H. Wannier, "An observation of heat sources placed at random on a surface," *J. appl. Phys.*, Nov. 1948, vol. 19, pp. 1020–1022.

The author considers a small plane interface between two solids of similar material. He assumes a highly irregular heat source function, $S(x, y)$, distributed on this interface, and then proceeds

to deduce certain properties of the source-difference function, $S(x, y) - S_0$, where S_0 is the area-mean of $S(x, y)$. It is first shown that if the first moments of the source-difference function vanish over the interface, then the area-mean potential (Newtonian) of this function also vanishes. Next, a fluctuation theorem is established which states that

$$4k^2[(\nabla T)_{11}{}^2]_{Av} = [(S - S_0)^2]_{Av}$$

where $(\nabla T)_{11}$ denotes the projection of grad T upon the interface and k is the thermal conductivity of the material. Finally, the author inserts certain qualitative relations into this fluctuation formula to deduce the effects of solid-solid friction.

Harold G. Elrod, Jr., USA

392. Hjalmar Granholm, "Heat insulation properties of hollow concrete blocks and perforated bricks" (in Swedish with English summary), *Trans. Chalmers Univ. Technol. Sweden*, 1948, no. 80, pp. 5–69.

The heat-insulating properties of perforated blocks are studied experimentally by means of an electric analog. Holes of the same form and disposition as those in the section of the block are cut in a sheet of aluminum foil, and the ratio of the electrical resistance with and without the hole is used as a figure of merit for the heat-insulating qualities of the block. An allowance for the heat transfer through the air in the cavities is made by leaving strips of foil transversally across the holes; the width of these is progressively reduced so that the effect of different heat conductivities of solid materials may be simulated.

For concrete blocks available in Sweden, figures of merit from 1.1 to 4.5 were obtained, but it is clear that by improved design, values up to six are quite possible. The best forms have long narrow rectangular holes, the long sides parallel to the outer wall surfaces, with six or more staggered rows across the section, with three or four holes in a row. Rounding the corners of the holes produces an appreciable reduction in the figure of merit.

Similar results, but with lower figures of merit, were obtained for sections common among perforated bricks. The results were compared with those obtained by tests on full-scale walls.

W. G. Bickley, England

393. L. I. Koodryashev, "Generalized integral relation for the thermal boundary layer and its application to the calculation of heat exchange" (in Russian), *Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR)*, Nov. 1, 1948, vol. 63, pp. 23–26.

The integral relation for the thermal boundary layer can be obtained, according to V. V. Goloobev, by multiplying the equation $\partial t/\partial \tau + W_x \partial t/\partial x + W_y \partial t/\partial y = a\partial^2 t/\partial y^2$ by $t^k dy$ (τ is the time; t the temperature) and integrating from 0 to δ_2 (thickness of the boundary layer; $t = t_0$, $\partial t/\partial y = 0$ at its outer surface). For $k = 0$ the energy-balance equation of the thermal boundary layer is obtained. It reads, in the stationary case:

$$\frac{d}{dx}\left[t_0(x) \int W_x dy - \int W_x t dy \right] = a \left(\frac{\partial t}{\partial y} \right)_{y=0}$$

(all integrals are from 0 to δ_2) and this form was used before by Kroozhilin and Shvab. If $k \neq 0$, still for the stationary case, the equation

$$\frac{d}{dx}(t_0^{k+1} W \delta^{**}{}_{2,k+1}) = \frac{dt_0^{k+1}}{dx} \int W_x dy - (k+1)a \int t^k \frac{\partial^2 t}{\partial y^2} dy$$

results, where, W being the scalar velocity,

$$\delta^{**}{}_{2,k+1} = \int \frac{W_x}{W} \left(1 - \frac{t^{k+1}}{t_0^{k+1}} \right) dt.$$

The assumption $k = 0$ meets most of the important applications. In this case,

$$\frac{d}{dx}(t_0 W \delta^{**}{}_2) = \frac{dt_0}{dx} \int W_x dy + a \left(\frac{\partial t}{\partial y} \right)_{y=0}. \qquad (*)$$

To obtain the heat-transfer coefficients α the author recommends (1) representing the velocity and temperature profiles in the boundaries by appropriate polynomials; (2) determining the thickness δ_1 of the fluid boundary layer by means of the integral relations of von Kármán and L. C. Leibenzon; (3) with the aid of δ_1 determining the thickness δ_2 of the thermal boundary layer from the integral relation (*); and (4) computing α as $2\lambda/\delta_2$. The author claims better efficiency for this method than for that using the momentum equations.

If polynomials of the fourth degree are used, the relations (**Nu** for Nusselt, **Pr** for Prandtl, **Re** for Reynolds number)

$$\mathbf{Nu} = C(\mathbf{Pr})^{1/3}(\mathbf{Re})^{1/2} \qquad (**)$$

with $C = 0.69$ is obtained. Pohlhausen, by direct integration of the momentum equation, found $C = 0.664$.

If $dt_0/dx \neq 0$, the method still works and yields:

$$\mathbf{Nu} = 0.345(\mathbf{Pr})^{1/3}(\mathbf{Re})^{1/2} {}_0\int^1 \phi^{1/3}(\xi) d\xi / \sqrt{\xi},$$

where $\xi = x/l$ and

$$\phi(\xi) = \exp[10.1 \, {}_0\int^\xi t_0'(\xi)/t_0(\xi) d\xi] \, {}_0\int^\xi \exp[-10.1 {}_0\int^\xi t_0'(\xi)/t_0(\xi) d\xi] d\xi$$

For $\phi(\xi) = 1$, the relation (**) is obtained.

The reviewer notes that the von Kármán-Pohlhausen results can be found in articles by L. G. Whitehead and N. A. V. Piercy [*Aircr. Engng.*, Jan. 1949, vol. 21, pp. 14–19].

D. Jacovleff, Belgium

Theoretical and Experimental Methods

(See also Revs. 293, 295, 329, 352)

394. Mario G. Salvadori and Kenneth S. Miller, "The mathematical solution of engineering problems," McGraw-Hill Book Co., New York, 1948. Cloth, 9.2 × 6. in., 245 pp., 121 figs., $3.50.

The title of this book does not make it quite clear that it is essentially a first course in calculus and analytic geometry. There are additionally thirty-four pages on Fourier series written in easily understandable language. A. W. Wundheiler, USA

395. C. J. Tranter, "The combined use of relaxation methods and Fourier transforms in the solution of some three-dimensional boundary-value problems," *Quart. J. Mech. appl. Math.*, Sept. 1948, vol. 1, pp. 281–286.

In this short paper a simple method of solving three-dimensional boundary-value problems by the relaxation method is given. All of the facts in the paper are well known but are organized in a new, interesting and useful way. Essentially, the author uses Fourier transforms to reduce the three-dimensional problem to n two-dimensional ones, each of which can be solved by the usual two-dimensional relaxation treatment. The three-dimensional solution is then the Fourier series with n coefficients or the Fourier transform involving an integral in n. This method will obviously be of practical importance only for such cases for which n can be taken small. This, therefore, involves some judgment on the part of the computer as to the convergence of the series (or integral) representation of his results. In an illustrative example the temperature distribution in a cube is computed

for the first four Fourier coefficients. This solution at the cube center is within two tenths of one per cent of the known analytical solution for this case.

Although a much larger number of coefficients may be needed in a more general case, the work involved must be compared to either an attempted analytical solution with subsequent difficult numerical evaluations or a direct three-dimensional relaxation of the problem. Neither of these alternatives is in general easy. The method proposed in this paper is by far the most practical computation scheme proposed to date for this type of problem.

Howard W. Emmons, USA

396. **M. Dumas and P. Maheu, "Statistical methods and their applications in the fields of industrial techniques (Les méthodes statistiques et leurs applications dans le domaine des techniques industrielles),"** *Mémor. Artill. fr.*, 1948, vol. 22, no. 2, pp. 413–480, and supplementary statistical pamphlet.

This article is in the nature of a digest of a book on statistics and its applications by M. Dumas. This reviewer was not favorably impressed by this work, both as regards choice of subject matter and its treatment. It is this reviewer's opinion that the subject is treated much more satisfactorily on a comparatively elementary level in recent texts by P. G. Hoel and S. S. Wilks. The authors of this article neglect much of the significant statistical work of the past twenty years, and it is difficult to see how the engineer could gain a proper perspective of the field of statistics from this article. The reviewer fails to understand the arguments used to justify the author's approach to the theory of statistical inference. It is certainly unconvincing and unsatisfying to be told that the a priori distributions chosen by the authors are accepted as a basis for operation when they do not meet one of the necessary conditions for a probability-density function, namely that it be integrable over the range in question. While such difficulties may seem minor to the authors, they leave serious doubts in the mind of the reviewer. Benjamin Epstein, USA

397. **Shigeo Uchida, "A new approximate solution in boundary-value problems by the use of orthogonal curvilinear co-ordinates"** (in English), *J. phys. Soc. Japan*, Jan.-Feb. 1948, vol. 3, pp. 33–40.

Two variants of Flügel's method of graphical integration of the partial differential equation [*Z. ges. Turbinenw.*, 1915, vol. 12, p. 73; 1919, vol. 16, p. 161] are presented. They consist in expressing the partial differential equation in terms of suitably chosen curvilinear co-ordinates and replacing it by an ordinary differential equation in a single variable, the remaining variables being regarded as parameters. They are applied to the compressible flow in a channel with circular walls, making use of bipolar co-ordinates. The theoretical results for the velocity distributions along the circular wall, both for the symmetrical and for the asymmetrical flows, are in good agreement with the author's experimental results. Isao Imai, Japan

398. **W. J. Duncan, "Assessment of errors in approximate solutions of differential equations,"** *Quart. J. Mech. appl. Math.*, Dec. 1948, vol. 1, pp. 470–476.

The author gives a short survey of work previously done by him on the assessment of errors in cases where the Green's function of the problem is one-signed. Furthermore he discusses the estimation of the errors of the step-by-step solutions of ordinary differential equations or sets of these with one-point boundary conditions, as well as the estimation of the errors of linear problems when an approximation of the Green's function is known. For the reader who is interested in these problems it may be of some help to look up the following references: (1) W. J. Duncan, "On the torsion of cylinders of symmetrical section" [*Proc. roy. Soc. Lond. Ser. A*, 1932, vol. 136, p. 95]; (2) "Torsion and flexure of cylinders and tubes" [*Rep. Memo. aero. Res. Counc. Lond.*, no. 1444, Feb. 1932]; (3) W. J. Duncan, "Technique of the step-by-step integration of ordinary differential equations" [*Coll. Aero. Cranfield Rep.*, no. 4, Feb. 1947; also *Phil. Mag.*, 1948, vol. 39, p. 493]. C. B. Biezeno, Holland

399. **Peter L. Tea, "A mechanical integraph for the numerical solution of integral equations,"** *J. Franklin Inst.*, May 1948, vol. 245, pp. 403–419.

The integraph is designed for evaluating integrals of the product of two functions, especially of the form $\int_a^b \phi(y)K(y \pm x)dy$ by using the transformation $\int_A^B \phi(y)dY(y \pm x)$ with $Y = \int K(y \pm x)dy$.

It can therefore be used for the solution of integral equations by successive substitutions, and also for evaluating the integrals occurring in the expansion of given functions in a series of orthogonal functions. Various examples of the application of the integraph are given. The accuracy of the results is considered.

Ludwig Biermann, Germany

400. **Pierre Vernotte, "Summation in finite terms of the most general Fourier series for the region outside the interval for which they were set up (Sommation, en termes finis, des séries de Fourier les plus générales, en tout point hors de l'intervalle où elles ont été formées),"** *C. R. Acad. Sci. Paris*, Nov. 8, 1948, vol. 227, pp. 966–968.

The author claims to give a step-by-step procedure for continuing a Fourier series outside the interval for which it was originally constructed. Such questions arise in a natural way in the solution of one-dimensional heat-flow problems, where the initial temperature distribution is specified over some subinterval of the x-axis. This paper gives no details of the procedure. Details are given in the author's "Thermocinétique" [*Publ. Sci. Tech. Ministre Air*, Série Grise, Fascicule 225].

Benjamin Epstein, USA

401. **J. A. Bronzo and H. G. Cohen, "A note on analog computer design,"** *Rev. sci. Instrum.*, Feb. 1949, vol. 20, pp. 101–102.

The authors observe that if the quantities $Y_i(t)$ satisfy a linear difference-differential equation in i and t with constant coefficients, so also do the quantities $Q_{ij}Y_j(t)$, where Q_{ij} is any non-singular constant matrix, and that if all solutions $Y_i(t)$ be stable so also are all solutions $Q_{ij}Y_i(t)$. In a practical case the transformed equations may be easier to mechanize in a computer, as is illustrated by an example. No method for finding a suitable matrix Q_{ij} is indicated. Horace M. Trent, USA

402. **Sandro Faedo, "On the Ritz method and methods based on minimum squares for the approximate solution of problems of mathematical physics (Sul metodo di Ritz e su quelli fondati sul principio dei minimi quadrate per la risoluzione approssimata dei problemi della fisica matematics),"** *R. C. Mat. Appl.*, June 1947, vol. 6, Ser. 5, pp. 73–94.

After a brief summary of the fundamentals of Ritz's method and of those based on the principle of least squares for the approximate solution of differential equations, the close relationship between both methods is pointed out.

The theorem, demonstrated by Picone in 1928, that for the second of these methods the magnitude of the error committed

with a given approximation $Y(x) = \Sigma_{k=1}^{n} a_{kn}\varphi_k(x)$ for $y(x)$ is of the order of $1 + \nu$ with respect to $1/n$, the coefficients of the differential equation being of class $C^{2+\nu}$, is stated. It is also pointed out that this theorem, when applied to Ritz's method, gives the value 3/2 for the magnitude of the error regardless of the value of ν.

It is then demonstrated that by taking in Ritz's method

$$\varphi_k(x) = (x - a)(b - x) \sin kn(x - a + \delta)/(b + 2\delta - a),$$

$(\delta > 0, k = 1, 2 \ldots),$

or some other analogous form instead of the usual

$$\varphi_k(x) = \sin kn(x - a)/(b - a),$$

a theorem, analogous to Picone's, is valid for Ritz's method.

Ernesto Saleme, USA

403. E. T. Goodwin and J. Staton, "Table of $\int_0^\infty e^{-u^2}du/(u + x)$," *Quart. J. Mech. appl. Math.*, Sept. 1948, vol. 1, pp. 319–326.

A table of values of the function

$$f(x) = \int_0^\infty \frac{e^{-u^2}}{u + x}\, du$$

has been computed for

$0 \leq x \leq 2$ in .02 steps,
$2 \leq x \leq 5$ in .05 steps,
$3 \leq x \leq 10$ in 0.1 steps.

An auxiliary function $f(x) + \log x$ has been tabulated to four decimal places for $0 \leq x \leq 1$ in 0.1 steps to facilitate interpolation for small values of x.

The expression

$$f(x) = \frac{\sqrt{\pi}}{2}\left(\frac{1}{x} + \frac{1}{2x^2}\right) - \frac{1}{2}\left(\frac{1}{x^2} + \frac{1}{x^4}\right)$$

can be used for values of x greater than 10, giving $f(x)$ to four places.

Different methods of computation were used for different ranges of values of x. For x less than 1 power series were summed, between 1 and 4 a differential equation satisfied by $f(x)$ was integrated numerically, and for x greater than 4 an asymptotic series was used. Details of the method of computation are presented.

E. G. Allen, USA

Acoustics

404. L. N. Liebermann, "The origin of sound absorption in water and in sea water," *J. acoust. Soc. Amer.*, Nov. 1948, vol. 20, pp. 868–873.

Measurements of the absorption of sound in the fresh water of a lake and in sea water were made at frequencies of 120, 230, 480, and 940 kc per sec. The method used was experimental determination of the variation of sound intensity with distance from resonant crystal sources. The results for fresh water are in substantial agreement with laboratory measurements made by various researchers at frequencies above 10^6 cps, on the basis of the usual deduction from the theories of Stokes and Kirchhoff that the absorption coefficient varies as the square of the frequency.

The author has recently determined the dilatational viscosity of water by measuring both the ratio of streaming pressure (due to sound waves) to radiation pressure, and the streaming velocity. The absorption computed from the combined effects of dilatational and shear viscosities is in good agreement with the measured values. Since heat conduction has negligible effect in pure water, the sound absorption is due entirely to viscous effects. In salt water, the absorption at low frequencies is much greater than in pure water, but the absorptions are similar at frequencies above 10^6 cps. The behavior of salt water can be explained by assuming a relaxation mechanism having a time constant of 10^{-6} sec. The author speculates that the relaxation involves the ionic dissociation of NaCl in solution. Thermodynamic calculations bolster this hypothesis.

Richard K. Cook, USA

405. James Woodburn, "Experimental determination of velocity of sound in superheated steam by ultrasonics," *Trans. Amer. Soc. mech. Engrs.*, Jan. 1949, vol. 71, pp. 65–72.

For over two decades, the acoustic interferometer has been employed widely for the precise determination of the velocity of sound in gaseous and liquid mediums. An application of this instrument to the study of superheated steam is the central theme of this article.

Great care was exercised by the author in the design of his equipment and in the taking of data. Velocity of sound determinations were accurate to about one part in two thousand. Other deduced quantities, such as the ratio of specific heats, had accuracies of about one part in a hundred, the drop in accuracy being attributed to difficulties in measuring the temperatures and pressures of the steam.

Data are presented covering the temperature ranges 300 F to 800 F at 24.69 psi and 400 F to 700 F at 99.69 psi. The results obtained are in close agreement with quantities calculated from the tables of Keenan and Keyes. There was no measurable dispersion in superheated steam for frequencies between 575,000 and 2,000,000 cps.

Horace M. Trent, USA

406. W. P. Mason and F. H. Hibbard, "Absorbing media for underwater sound measuring tanks and baffles," *J. acoust. Soc. Amer.*, July 1948, vol. 20, pp. 476–482.

It is desirable to calibrate underwater sound transducers in the laboratory or factory. This can be done only if the tank in which the transducer is measured has sound-absorbent walls. The authors designed such walls by introducing at the wall a liquid having (1) the same characteristic impedance as water, and (2) a much greater viscosity than that of water. The absorption was increased by introducing metal screens into the liquid. A wall about 6 in. thick composed of 100-mesh screens spaced 0.2 in. apart and immersed in castor oil reflected not more than about 1 per cent of the incident sound energy (frequency range 10 to 50 kc per sec). Rubber having the same impedance as water was used to separate the castor oil from the water. The mechanical details of construction for a tank having such walls, and holding about 10 cu ft of water, are given. The calibration of a transducer in the tank agreed with the open-water calibration for those directions for which the response was not more than 25 db below the maximum response.

Richard K. Cook, USA

407. W. P. Mason and H. J. McSkimin, "Energy losses of sound waves in metals due to scattering and diffusion," *J. appl. Phys.*, Oct. 1948, vol. 19, pp. 940–946.

The authors treat the loss in energy of longitudinal and shear sound waves caused by scattering and diffusion due to the individual grains of a multicrystalline rod of metal. The frequencies involved are from 2 to 80 megacycles per sec. Scattering losses obeying Rayleigh's law (which states that the attenuation is proportional to the grain volume and to the fourth power of the fre-

quency) are observed for frequencies for which the grain size is less than a third of the wave length, while diffusion occurs at frequencies for which the wave length is less than one third of the grain size. The attenuation due to diffusion is controlled by the mean free path, which here means the distance the wave can travel before suffering reflection or refraction. It is shown that diffusion losses are inversely proportional to grain diameter, which is in reasonable agreement with experiment.

Since the grain directions are randomly distributed and since the elastic constants vary with direction, there is a discontinuous change in elastic constant across the grain boundaries resulting in reflection, which is the reason for the scattering loss. Scattering factors for longitudinal and shear waves for both cubic and hexagonal crystalline metals are computed by finding an average spatial value of the longitudinal and elastic shear constants, assuming that all orientations of the grains are equally probable. A significant correction to this computation is given in an errata [*J. appl. Phys.*, Feb. 1949, vol. 20, p. 228]. Agreement between theory and experiment is good. The only metals having a low scattering loss are aluminum, magnesium and tungsten. In particular, magnesium has a very low shear loss.

Albert London, USA

408. **R. J. Urick, "The absorption of sound in suspensions of irregular particles,"** *J. acoust. Soc. Amer.*, May 1948, vol. 20, pp. 283–289.

The author measures the absorption of sound in suspensions of irregular particles in liquids, and compares the measurements with the absorption computed for spheres. The measurements were made with a pulse reflection technique at frequencies between 1 and 15 megacycles per sec. The suspensions included kaolin (flat plates, median size 0.9 micron), quartz sand (irregular, median size 2 2 microns), and lycopodium spores (uniform diameter of 30 microns) in water. The effect of viscosity was studied by means of kaolin suspensions in glycerin-water solutions, and in methyl cellulose solutions. The absorption was computed by addition of the energy scattered by spherical particles and the energy absorbed due to viscous drag between the spheres and the liquid.

The theory predicts that (1) the absorption coefficient (per cm) is directly proportional to the concentration of the suspended material; (2) the absorption at 1 megacycle per sec decreases as particle size is increased above 1 micron for 1 per cent (by volume) suspensions in water; (3) the absorption remains substantially constant even when the viscosity of the liquid is varied over a range of 200 to 1; (4) the absorption is approximately proportional to the frequency. The theoretical results on spheres are in good agreement with the measurements on irregular particles.

Richard K. Cook, USA

Ballistics, Detonics (Explosions)

(See also Rev. 378)

409. **E. K. Rideal and A. J. B. Robertson, "The sensitiveness of solid high explosives to impact,"** *Proc. roy. Soc. Lond. Ser. A*, Dec. 7, 1948, vol. 195, pp. 135–150.

Oscillographic methods have been used to examine the periods between impact and explosion, and impact and rebound, in the explosion of several solid high explosives confined between steel surfaces and subjected to the blow of a falling weight. Impact-explosion times up to several hundred microseconds were observed, the time increasing with diminishing impact sensitiveness of the explosive and with lower impact energies. The impact-rebound time always exceeded the impact-explosion time in the same experiment. The effect of foreign substances on the times was examined; the time lag before explosion was diminished by adding quartz but could not be increased by addition of soft materials. The mean velocity of blast-wave propagation from explosion increased with the impact-explosion time.

The conditions for the growth of a hot spot in a volume of explosive are formulated in terms of thermal and kinetic data, and critical hot-spot temperatures computed for several explosives show a correlation with impact sensitiveness. The calculated time of hot-spot reaction is much less than the impact-explosion times. Viscous flow under high pressures is postulated as a possible mechanism of hot-spot formation, the impact-explosion period being mainly the time required for the pressure to rise to the necessary high value. A direct demonstration of the occurrence of high-flow velocities under impact was made.

J. Howard Childs, USA

410. **M. Garnier, "Ballistics of a spinning projectile (Balistique d'un projectile gyroscopé),** *Mémor. Artill. fr.*, 1947, vol. 21, no. 1, pp. 113–208.

This paper treats the problem of computing the motion of a spinning projectile, as contrasted with the classical ballistic problem of the motion of a heavy particle subjected only to drag and gravity forces. The author is a pioneer in ballistic computations, being responsible for developing with two colleagues the G. H. M. (Garnier-Haag-Marcus) method in general use in France for solving the classical ballistic problem.

The author first assumes the elements of the classical trajectory of the projectile treated as a heavy particle as given quantities, and studies the motions of the axis of the projectile. The angle of yaw is assumed small and specified by a single complex variable Z in a plane tangent to a sphere of unit radius about the center of gravity at its point of intersection with the trajectory. This variable satisfies a differential equation of the second order with variable coefficients and with a right-hand term. A method of computation similar to the G. H. M. method is used to solve this equation.

Knowing the angle of yaw, the normal and Magnus forces are estimated, and the perturbing effects on the classical trajectory are computed. A numerical example is given.

See also K. L. Nielsen and J. L. Synge ["The motion of a spinning shell," *Quart. appl. Math.*, 1946, vol. 4, p. 201], where further references to American work on the subject can be found.

Hugh L. Dryden, USA

Soil Mechanics, Seepage

411. **L. Marivoet, "A simplified method for computing the bearing capacity of the soil supporting footings or piers"** (in English), *Proc. Sec. int. Conf. soil Mech. found. Engng.*, 1948, vol. 1, pp. 78–86.

Graphs are presented of the bearing pressure of continuous footings as a function of the depth and breadth of the footing, and of the angle of internal friction. The calculations are based on an empirical extension of the Prandtl formula for a cohesionless material. Graphs are also given for the settlements which are calculated by Terzaghi's method of influence coefficients.

Rodney Hill, England

412. **Edgar Schultze, "Recent investigations on the bearing capacity of soil (Neuere Untersuchungen über die Tragfähigkeit des Baugrundes),"** *Bauplan. Bautech.*, 1948, vol. 2: Mar., pp. 73–79; May, pp. 147–150.

The article gives a summary of theories of soil-bearing capacity. The admissible stress is dependent not only upon the ultimate

bearing capacity of subsoils but also on the admissible amount of consolidation. The shape of the sliding surface may be assumed or computed on the basis of stress-distribution. The factors influencing the bearing capacity of subsoils are the unit weight of subsoil, pressure of soil-surcharge, cohesion, angle of repose, width of foundation, and depth of foundation plane below the surface. After dealing with the simplest case of narrow-strip footings subjected to a linear unit loading, the author considers stress distribution under rectangular footings. Finally, the resistance to oblique loading and the condition of tilting is discussed, with the conclusion that no acceptable formula exists for the computation of soil resistance against oblique loading.

It is advisable to compute consolidation values (except for the case of narrow or deep foundations) which might be more relevant to the safe loading of subsoils (chiefly in the case of wet clay and loose sands) than the danger of rupture.

Ch. Széchy, Hungary

413. F. Szelagowski, "On the depth of foundation" (in English), *Proc. Sec. int. Conf. soil Mech. found. Engng.*, 1948, vol. 1, pp. 115–119.

The author combines Coulomb's and Rankine's formulas for soil-shear resistance with Michell's theory of vertical and horizontal stresses in the soil under a given load. By considering a prismatic element under the foundation and assuming that the shear resistance of the material is not exceeded, a bearing pressure may be found as a function of the depth of the footing and the angle of internal friction and cohesion. The weight of the soil is eliminated by expressing the bearing pressure as the height of a column of soil of density equal to that considered.

The author considers only active earth pressure, in view of the displacements required for the existence of passive earth pressure. Accordingly, the whole question of the actual line of rupture is bypassed as being too complex for analysis. A critical study of a number of theories using the line of rupture is included.

Robert Quintal, Canada

Geophysics, Meteorology, Oceanography

(See also Revs. 296, 321, 361, 387)

414. H. P. Schmitz, "An extension of present theories concerning the vertical turbulent heat flow (Eine Erweiterung der bisherigen Theorien des vertikalen Turbulenzwärmestroms)," *Z. Met.*, Nov. 1947, vol. 1, pp. 422–430.

This is an extensive investigation of the problem of vertical transport of heat and some other scalar quantities in the atmosphere when no vertical mean motion is present. The transport in the vertical direction is proportional to the correlation between the vertical turbulent velocity component ζ, and the fluctuation of the quantities in question: $\delta\rho$ density, δp pressure, δT temperature, $\delta\vartheta$ potential temperature (the δ represents fluctuation about the mean value).

The author shows that Wilhelm Schmidt's assumption of $\overline{\zeta_z \delta p} = 0$ along with the standard assumption of zero vertical mass-transport ($\overline{\zeta_z \delta\rho} = 0$) leads to zero heat transport, contrary to Schmidt's result. With the help of thermodynamic relations it is shown that the correlations of the various scalar properties with the vertical turbulent velocity have symmetrical expressions in terms of the mean vertical gradients:

$$\frac{\overline{\zeta_z \delta p}}{\partial p/\partial z} = \frac{\overline{\zeta_z \delta\rho}}{\partial\rho/\partial z} = \frac{\overline{\zeta_z \delta T}}{\partial T/\partial z} = \frac{\overline{\zeta_z \delta\vartheta}}{\partial\vartheta/\partial z}$$

The new quantity introduced is $\mu(\vartheta)$. The vertical displacement of the ϑ = const layers from their mean position is called $\xi_z(\vartheta)$. The correlation of this vertical displacement with the vertical turbulent velocity component is $\mu(\vartheta)$, and all the other correlations describing vertical transport are expressed in terms of μ and the gradients of the appropriate mean properties.

The method is finally used to discuss in detail the transport of heat, potential temperature, entropy and enthalpy between horizontal planes, with or without heat sources or sinks present.

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415. Philip Duncan Thompson, "The propagation of permanent-type waves in horizontal flow," *J. Met.*, Aug. 1948, vol. 5, pp. 166–168.

The final frequency equation presented in this paper is essentially correct and of considerable importance. The work leading to it unfortunately cannot be regarded as correct except under very restricted conditions which are entirely obscured by the presentation. It can be shown that this method involves the assumptions of small perturbations, of an eastward wave speed which is always smaller than the eastward speed of the basic current, and of a basic current profile which has only one specialized form. The solution is not, therefore, nearly as general as implied in the paper. For a more general and complete attack on this problem the reader is referred to a paper by Kuo ["The dynamic instability of two-dimensional nondivergent flow in a barotropic atmosphere," *J. Met.*, 1949, vol. 6 (to be published)] which has served as basis for a derivation and discussion of the frequency equation by J. S. Malkus ["Some features of disturbed and undisturbed weather in the trade-wind region," doctoral dissertation, University of Chicago (to be published)].

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Marine Engineering Problems

(See Revs. 297, 298, 299)